# Analysis of the stability of masonry-faced earth retaining walls

## Prepared for Dr R Kimber, Research Director, Transport Research Foundation

K C Brady and J Kavanagh

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# Executive Summary


This report summarises the results of surveys of the stock of masonry-faced earth retaining walls along the motorway and trunk road network in five local authorities in England and Wales. The mean replacement value and annual maintenance expenditure in these areas were estimated to be £1.15M per km and about 0.75 per cent respectively. From these figures, it is estimated conservatively that there are about 140 km of masonry-faced retaining walls on the trunk roads in England and Wales having a replacement value of about £150M. From the results of a partial census, it is estimated that there are some 9000 (± 2000) km of retaining walls on the road network in Great Britain having a replacement value of between £7.2 ± 1.6 billion, with about 85 per cent of this attributable to masonry-faced earth retaining walls.

It is concluded from the relatively low level of maintenance expenditure that, overall, the stock is performing well and much of it will have a considerable residual life. Despite this satisfactory state of affairs, many masonry-faced walls would not have an adequate factor of safety as required by current design codes. The report reviews the modes of failure of masonry-faced retaining walls, the factors that affect their stability and the methods used to quantify stability.

The report presents and discusses the results of discrete element analyses undertaken on four full-scale walls constructed at Kingstown, now Dun Laoghaire, in Ireland, by Burgoyne in 1834. Two of the walls collapsed when filling reached a height of about 5.2 m, whilst the other two were stable at their full height of 6.1 m. Using the best estimates for the input variables, the essential behaviour of the four walls was reproduced by the analyses. The estimated deflections of the two stable walls did not agree particularly well with the reported values, but this is understandable given the uncertainty regarding the stiffness of the interface between the blocks making up the wall. For the two walls that failed, the analysis reproduced the failure height and the observed pattern of deformation prior to, and following, collapse.

Further analyses were undertaken to investigate the effect of particular variables on the performance of these test walls, including the properties of the interfaces between the blocks, the shape of the blocks and the joint pattern of the blocks making up the wall. For the test walls, the results of these analyses showed that sliding movements between the blocks were relatively small compared to those generated by overturning of the base about its toe. The successive introduction of horizontal or sub-horizontal joints marginally increased outward movement, but the introduction of a vertical joint running parallel to the face of a wall either had little effect or it led to failure by overturning.

An examination of the methods commonly used to define overturning stability shows that they do not have unrestricted applicability. The methods are somewhat arbitrary as are, therefore, the minimum safety factor values prescribed for them in a design or assessment code. Furthermore, the assumption that a jointed wall acts as a monolith up to the point of collapse is non-conservative. This is because with a monolith overturning can only occur about its base but with a jointed wall it can occur on a plane inclined to the base so that the full weight of the wall does not resist the overturning forces.

The report considers the role of numerical methods of analysis for the design and assessment of masonry-faced earth retaining walls. Because of the cost of the exercise, at this time, it seems unlikely that a numerical method will be used as a matter of routine for assessing such walls but usage here will inevitably increase with the reduction in the cost of computation. But it seems that such methods should have a bigger role to play in the design of retaining walls and foundations.

# 1 Introduction

Masonry-faced earth retaining walls are commonplace alongside highways in the more hilly and mountainous areas of the UK. Most of them were built in the 19th and early 20th centuries as dry-stone walls. According to Jones (1979), '*the dry-stone retaining wall is the most common feature in the landscape*' of the Yorkshire Dales: here stone is plentiful and through, and following, the Industrial Revolution many roads were built on sidelong ground. Jones has also provided details of dry-stone retaining walls, the problems associated with them and their maintenance (1990 and 1992).

Most of the in-service dry-stone walls are true to line and level, have not required any substantial maintenance, and show no signs of instability. This might not always have been the case, but a century or more of usage has weeded out most of the walls that were of inadequate construction whilst marginally stable structures have been improved over the years - typically by pointing or pressure grouting of the face and/or injection behind. Thus, in the UK, the stock of dry-stone retaining walls and their derivatives is, by and large, performing satisfactorily and is likely to do so for some time to come although some walls are undergoing a gradual decline.

The simple explanation for the continued stability of such walls is that the forces acting to disturb them are lower than the stabilising forces, but analysis is complicated because a number of factors are involved. Current design codes preclude the inclusion of some optimistic but, nevertheless, real factors and so numerical assessments undertaken using these codes would lead to the erroneous conclusion that many of these walls are unsafe and, by implication, that they need to be replaced. Clearly, no matter what the results of such a paper assessment indicate, it is of great economic importance that perfectly adequate structures remain in service. Better methods of assessment are, therefore, required to ensure that (a) expenditure on replacing perfectly adequate structures is minimised (b) structures at risk of collapse are identified, and (c) appropriate remedial works are defined and undertaken to maintain the stock in good order.

A number of studies on the design and performance of masonry-faced earth retaining walls have been completed by TRL Limited. These have involved a review of the stock of such walls and of the means for assessing their stability. Most of this work has been undertaken for the Highways Agency (HA) and the results are in the public domain but the most recent study was undertaken for the Transport Research Foundation (TRF), as part of its reinvestment research programme, and the results of this have not previously been published.

To provide an essential background to the subject, this report provides information on the work undertaken for the HA. However the main concern of the report is the latest study on the analysis of the stability of jointed walls.

# 2 The stock of structures

### 2.1 UK trunk road and motorway network

Over the period 1995-97, surveys were commissioned in five local authorities in England and Wales. The authorities involved were the Counties of Derbyshire, Gwynedd (pre-1996 boundaries), North Yorkshire and Powys, and the Municipal Borough of Calderdale.

The summarised data in Table 1 show that the records of 1474.5 km of trunk roads were examined in the surveys: this represents almost 16 per cent of the length of trunk road in England and Wales (DoT, 1996). The total length of masonry-faced retaining walls over this length of trunk road was 92.7 km. Most of the data were extracted from the Department of Transport (DoT) report forms (TRRM 2/88) for the structures. Such forms cover '*retaining walls over 1.5 m height from finished ground level in front to the top of the wall*.' It is unclear as to whether or not the height includes the parapet, but this does not affect the data appreciably.

Although there are differences in the percentages of the various types of masonry-faced retaining walls in the five inventories, there is good agreement on the percentage of walls in the dry-stone/mortared-stone category. In four of the five, the percentage varied narrowly between 88 and 91 per cent, whilst it was 73 per cent in the other. The great majority of these walls are likely to have begun life as dry-stone walls: following pointing or pressure grouting they are now categorised as mortared-stone. Most of these dry-stone/mortared-stone walls will not have a sufficiently high factor of safety as calculated and required by current design codes.

The TRRM 2/88 report form requires an estimate of the value of the structure concerned. The estimated value of the 92.7 km length of masonry-faced earth retaining walls was a little over £107 M - that is, an average of about

**Table 1 Summarised data of surveys (from O'Reilly *et al*., 1999)**

| | *Local authority area* | | | | | |
|---|---|---|---|---|---|---|
| *Item* | *Derbyshire* | *Gwynedd* | *North Yorks* | *Powys* | *Calderdale* | *Total* |
| Length of trunk road within authority (km) | 190.2 | 379.8 | 444.5 | 431.4 | 28.6 | 1474.5 |
| Length of masonry-faced retaining wall (km) | 37.4 | 31.3 | 6.4 | 8.2 | 9.4 | 92.7 |
| Estimated replacement cost of masonry-faced retaining walls (£Million) | 37.8 | 44.8 | 6.6 | 8.5 | 9.6 | 107.3 |
| Average annual cost of maintenance and renewal of masonry-faced retaining walls (£Million) | 0.18 | 0.39 | 0.05 | 0.06 | 0.13 | 0.81 |

£1.15 M per km. This estimate is based on actual replacement costs in the five local authorities and so it includes some element for traffic management works but because it does not include traffic delay costs it underestimates the total replacement cost. For the sake of simplicity, in this study, asset value and replacement cost have been taken to be synonymous.

The recorded annual expenditure on the maintenance and renewal of the walls, averaged over at least the five previous years, was a little over £0.8 M - representing just 0.75 per cent of the replacement cost of the walls. This shows that the stock of walls has a considerable residual lifetime. The replacement costs are summarised in Table 2: the higher costs of the walls in Gwynedd probably reflect the more rugged terrain of North Wales. The relations between cost and wall height (h) were assumed to be linear, but the cost of constructing retaining walls probably varies with $h^2$. A linear relation is likely to underestimate the replacement costs of the higher walls, but overestimate the costs of the lower ones - which are more prevalent.

According to the Scottish Office there are few masonry-faced earth retaining walls on their recently improved trunk road system. The same situation pertains in parts of Wales and England where the construction of motorways, bypasses and the improvement of lengths of trunk road have eliminated many such walls. Nonetheless such walls are commonplace where realignment of the existing highway is impractical or prohibitively expensive, such as in the hilly parts of Derbyshire. Here the substantial lengths of existing dry-stone walls must either be retained or renewed as they now exist.

It can be estimated, reasonably conservatively, that there are some 140 km of masonry-faced retaining walls on trunk roads in England and Wales having a replacement cost of about £150 M and an annual maintenance and renewal expenditure of about £1.1 M. It seems unlikely that the length of such walls on trunk roads exceeds 200 km - with the corresponding replacement cost and annual expenditure being £230 M and £1.7 M respectively.

## 2.2 Road network of Great Britain

According to the returns from a Department of Transport (DoT) census of highway structures (DoT, 1987) there were 5394.7 km of retaining and supporting walls on highways in Great Britain. All seven metropolitan authorities in England provided information, but 27 of the 56 other highway authorities did not. Simple proportioning would suggest a total of about 9000 km of retaining walls. There must be considerable reservations about the accuracy of such a figure, but the true figure is likely to be within the limits of 9000 ± 2000 km.

According to this census about 50 per cent of the retaining walls were of dry-stone construction, 35 per cent of masonry and the remaining 15 per cent of concrete. On this basis, the application of current design rules for assessment purposes might be problematic for about 85 per cent of the stock of walls in Great Britain.

As mentioned above, the results of the surveys indicated that the mean replacement cost of the masonry-faced walls on trunk roads was about £1.15 M per km: this corresponds to an average retained height of about 2.4 m. This mean value will be on the high side for the road system as a whole because alignment standards for most of the system are less severe than for the trunk road network. A replacement cost of about £0.8 M per km would appear more appropriate: this is equivalent to a mean retained height of about 1.6 m and is close to the figure used by North Yorkshire for retaining walls on their county roads. The cost of refurbishment and repair would be less, but this presupposes that maintenance is undertaken so that the walls are strengthened and improved before they become unsafe or collapse. Unfortunately the present level of maintenance funding is such that on many of the less important roads nothing is done until collapse has occurred. Indeed there are times where the lack of funding to repair a dangerous wall has led to the partial closure of a trunk road: for example, along a section of the A5 in North Wales where the carriageway was reduced to a single-lane with traffic light control, and on the A82 by Loch Lomond in Scotland.

From the above, the replacement cost (value) of the 9000 ± 2000 km of retaining walls on the roads of Great Britain is likely to be £7.2 ± 1.6 Billion, with perhaps up to 85 per cent attributable to dry-stone walls and their derivatives. Putting this in context, the total figure represents between about 15 to 20 per cent of the total value of the bridge stock.

## 2.3 International

Dry-stone walls are evident across Europe, but the literature on their extent and condition is sparse. It is difficult to form an overall view from the fragmentary evidence but, as stated by O'Reilly *et al.* (1999), it would not be unreasonable to accept that there is at least twice the length of masonry-faced retaining walls on roads in Europe as there is in Great Britain. That is, 18000 km or so - but it would be unsurprising if the actual length turned out to be 2 to 3 times that figure.

**Table 2 Costs of replacing masonry-faced retaining walls (from O'Reilly *et al.*, 1999)**

| *Retained height (m)* | *Replacement cost (£/m run)* | | | | | *Average (£/m run)* | *Range (%)* | |
|---|---|---|---|---|---|---|---|---|
| | *Derbyshire* | *Gwynedd* | *North Yorks* | *Powys* | *Calderdale* | | | |
| 1.5 | 810.5 | 907.5 | 762.0 | 682.5 | 691.5 | 770.8 | +17.7 | -11.5 |
| 3.0 | 1436.0 | 1605.0 | 1350.0 | 1365.0 | 1383.1 | 1427.8 | +12.4 | -5.4 |
| 4.5 | 2061.5 | 2302.5 | 1938.0 | 2047.5 | 2074.6 | 2084.8 | +10.4 | -7.0 |
| 6.0 | 2687.0 | 3000.0 | 2526.0 | 2730.0 | 2766.2 | 2741.8 | +9.4 | -7.9 |

There is a good deal of information available on old masonry retaining walls in Hong Kong - many of which are of dry-stone construction. For example, according to Chan (1996) there are some 1750 masonry retaining walls over 3m high, but usually less than 10 m high, which were constructed from 1840 onwards into the early part of the last century. Much of the housing in Hong Kong is built on terraces formed on steep hillsides and failures of the retaining walls supporting these terraces have often led to loss of life. For example, the collapse at Kwun Lung Lau of a 700 to 800 mm thick masonry-faced retaining wall with a maximum height of 10.6 m resulted in 5 fatalities: details of the failure are given in a report of the Geotechnical Engineering Office (1994) and by Wong and Ho (1997).

Kim (1975) describes the construction of masonry-faced retaining walls in Korea, and Chan (1996) provides similar information on walls in Japan. According to Gupta and Lohani (1982), in northern India dry-stone walling is normally used for retaining walls up to 4 m high in sidelong ground.

### 2.4 Strategy for maintenance and renewal

As stated by O’Reilly *et al. (*1999),

> ‘*Routine maintenance activities are humdrum, sporadic, variable and difficult to resource efficiently: this does not of course mean that overall they are not the most economical way to retain the stock of masonry-faced retaining structures in a serviceable condition.*’

Pointing, pressure pointing and grouting behind the wall are common methods for improving the stability of dry-stone walls. Where land is available, buttresses, earth embankments and thickening of the wall have also been used, and where not ground anchorages and soil nailing have been installed; for example, respectively, on the A487 Tal Y Llyn Pass (see Flower and Roberts, 1987) and on the A5 at Nant Ffrancon (see Johnson and Card, 1998).

Where repairs cannot be carried out safely, or the wall has collapsed, then a replacement structure would normally be required: in some cases a masonry-faced wall would be required to be in keeping with its surroundings.

As shown by O’Reilly *et al.* (1999), a policy of replacing the stock, over a period of up to 100 years or so, that do not comply would be substantially more expensive than the current regime of maintenance and renewal. A policy of replacing all non-compliant walls would ensure that the stock complied with current standards but this would be a rather wasteful approach. The other extreme policy is to allow walls to collapse (or nearly so) before taking action, but for safety reasons this is unlikely to be acceptable even if it were economically expedient in the short term. The pragmatic approach is to treat the stock of structures requiring upgrading using a combination of strengthening and, where necessary, replacement.

The qualitative assessment procedures currently used in the UK are sensible, and the policy of maintenance and renewal is cost effective: this is described in Section 4. However there is room for improvement. Firstly, more information is required on the stock of structures on the entire road system, and on their rates of deterioration and collapse as well as on maintenance and replacement costs including the cost of traffic delays. Secondly, there is a need to develop methods for better quantifying the stability of in-service structures.

## 3 Failure modes

The failure modes of a masonry-faced earth retaining wall that need to be considered are:

1 Collapse by overturning.

2 Sliding along a slip surface that cuts through the wall.

3 Sliding along a slip surface that runs behind and beneath the wall.

4 Sliding along the base of the wall.

5 Excessive settlement of the foundation to the wall and/or the retained backfill.

6 Excessive distortion of the wall face (although this is likely to be the result of one or more of the above).

The first four listed are ultimate limits states (ULS), whilst the last two are serviceability limit states (SLS). The subject of this report is the assessment of stability, and so it is principally concerned with ULS. The definition of ‘excessive’ movement might be based on aesthetics as much as engineering considerations, and so it will vary from site to site depending upon factors such as the location and purpose of the structure. It is the case that many old masonry-faced walls will show fissures and bulging, but these do not necessarily mean that the wall is unstable. Nonetheless, such features are often the prelude to the occurrence of an ULS. Analyses to explain the bulging, toppling and shear failures occurring in dry-stone walls have been put forward by Cooper (1986).

It should be appreciated that, once successfully constructed, the stability of a wall will not change unless there is a change in either the loads acting on the wall or the strength of the structure. It seems reasonable to suppose that a structure that has stood for 100 years or more will have experienced all but the more improbable loads it could ever face. Exceptions might include walls subject to earthquake loading, and those supporting high live loads from traffic - such loads have increased, and might increase further, with time. Construction activity close to a wall can, of course, substantially change the loading conditions. Instability might also be induced by an increase in the pressures acting on the back of the wall and by a reduction in the strength of the backfill, facing or foundation soil.

### 3.1 Factors affecting stability

#### *3.1.1 Water*

Second only, perhaps, to the destabilising effects of nearby construction, the effects of water, either by the pressure it creates behind a wall or by weakening the backfill, is the most important factor tending to induce instability in an earth retaining structure. As constructed, dry-stone retaining walls are quite porous and any free water behind them has no difficulty in escaping; as a result water pressures acting on the wall are negligible. But, unless care is taken to maintain adequate drainage paths through a repaired section, this satisfactory situation can be compromised by techniques such as pointing, pressure pointing, and grouting. Also, the migration of fines from the backfill might, over time, clog the interstices in a dry-stone wall and give rise to localised bulging, instability and collapse. Ensuring that conditions are as dry as possible behind earth retaining walls and maintaining such conditions is of the utmost importance for ensuring the stability of such structures.

The ability of uncemented soils to resist shear stresses at zero total stress, generally called ‘cohesion’, depends on soil suction; i.e. the tensions within the menisci of moisture in the soil pores. The phenomena is described widely in the literature - see for example Croney *et al*. (1952), Fredlund and Rahardjo (1993) and, perhaps more accessible, Ridley and Brady (1997). At a particular horizon, the suction developed within a soil is affected by (a) the proximity of free water, particularly a permanent source of water and (b) the stress history of the soil. Thus the suction in an over-consolidated clay increases through unloading, for example by excavation: as discussed by Vaughan (1994) it might take a century or more for equilibrium to be reached in the soil behind a retaining wall in a cut slope. Suction can be generated by evapo-transpiration and, because the face of a dry-stone wall is highly permeable, suction might, therefore, be developed in the backfill. However, because the backfill to dry-stone walls is almost certainly coarse-grained (see below) the maximum suction developed might be moderate, and even this level might not be sustained throughout the year. It would seem, therefore, that suction does not have a major influence on the stability of a dry-stone retaining wall, but further fieldwork would be welcome to quantify its effect.

The analysis of the failure at Kwun Lung Lau indicated that an apparent cohesion of 8 to 10 kN/m$^2$ was needed to maintain stability there (Geotechnical Engineering Office, 1994): this is equivalent to a suction of about 13 kN/m$^2$. (Simple calculations show that a soil exhibiting a cohesion of 1 kN/m$^2$ could stand with an exposed face to a vertical height of between 1 and 2.4 m.)

### *3.1.2 Properties of the retained ground*

Although weak but intact rocks can stand vertically and so do not impose any loads on walls in front of them - whose main function is to inhibit weathering - even quite a hard rock can impose lateral pressures when it is fractured or has outward-dipping fracture planes.

For naturally occurring siliceous sands and gravels the minimum angle of shearing resistance, f, can conservatively be taken to be 30° (BS 8002: 1994). However, through the effects of dilation, it can be up to about 17° higher depending on the angularity, grading and density of the material. These values of $\varphi$ apply to randomly placed materials, but dry-stone walling amply demonstrates that systematically stacked blocks of rock, sandbags, and gabions can stand vertically to a considerable height. Furthermore, the artisans forming such retaining walls will often have placed the residues knapped from the facing stones in regular fashion behind the wall, thereby reducing backfill pressures, improving drainage and thereby enhancing the stability of the wall. For example, inspection of repair works to dry-stone retaining walls in Derbyshire has shown the stoney backfill to the original walling to be standing at 60° or more to the horizontal.

Although the shear strength of granular soils is largely unaffected by moisture content, the strength of clayey soils can be substantially affected. It is pertinent to note here that, despite an investigation of likely candidates, the surveys in the UK did not identify any dry-stone retaining walls as having a distinctly clayey backfill.

As well as exerting outward pressures on a retaining wall, the backfill also generates frictional forces on the back of a wall. In the normal course of events the backfill will settle more than the facing and so these forces act in a downward direction on the back of the face and thereby increase stability. Overturning of the face will also generate downward acting frictional forces. On the other hand, on poor foundations where the wall settles more than the backfill these forces are upwards and, therefore, destabilizing.

### *3.1.3 The wall face*

Clearly, the strength of a dry-stone wall is a function of the properties and interaction of the blocks making up the wall face. Weathering of the blocks might lead to a loss of integrity sufficient to lead to local deformation and perhaps, ultimately, collapse. This issue of durability should only be a problem where, perhaps for environmental or aesthetic reasons, a like-for-like reconstruction is required.

Walls formed from squared, slabby blocks of stone are more stable than ones formed of less regular and more random stonework, but there is little information on the effect of the joint pattern on the stability of a dry-stone wall. The effect of jointing was investigated as part of the study undertaken for the TRF: this was limited to plane strain analysis but it was appreciated that local stability, for example the formation of bulging, is governed, in other than random rubble walls, by three-dimensional considerations. The size of the blocks making up a wall is clearly important: a progressive reduction in size will ultimately produce a pile of grains that would only be stable at its angle of repose.

As with any gravity retaining structure, which depends on its weight to resist the ground thrusts acting upon it, the geometry of a dry-stone retaining wall can have a significant bearing on stability. Indeed this was the focus of the experimental studies carried out in the first half of the 19th century by the Royal Engineers: see Anon (1845) and Burgoyne (1853). The former describes the experiments by Lieutenant Hope at Chatham that, following a series of model tests, culminated in full-scale tests on three walls built of brick without mortar in the joints. Details of the latter are provided in Section 6.

# 4 Qualitative assessment

The current requirements for the inspection of all types of structure on the UK trunk road and motorway network are set out in BD 63 (DMRB 3.1.4) and as amplified in BA 63 (DMRB 3.1.5). These define the types of inspections and the intervals between them. Methods of classifying the extent and severity of defects and the priority for repair work are provided: as these are couched in general terms much depends on the skill and experience of the inspector.

The requirements for assessing highway structures are defined in BD 21 (DMRB 3.4.3) and advice on the assessment of retaining structures of all kinds is given in BA 16 (DMRB 3.4.4). The latter document is couched in general terms, but the advice on the assessment of dry-stone retaining walls given in Annex H is more specific and focuses on the issues to be resolved during an assessment process. Paragraph 8.5 of the former document states:

> '*if a foundation, retaining wall or a substructure shows no signs of distress, if there is no evidence of scour either externally or internally, and if no significant increases in load are envisaged, then the foundation, retaining wall or sub-structure may be assumed to be adequate and no further assessment is necessary*'.

This encapsulates the essence of the qualitative assessment process. And, importantly, in recognising that structures can be assessed qualitatively, by considering the condition of the structure and the significance of any defects, the inspection and assessment processes are contemporaneous and inextricably linked.

The question arises of what needs to be done where one, or more, of the above qualifications is not met. Of course, not all non-compliant structures need be replaced, even if that were possible. Unless its original condition can be restored readily and cheaply, a quantitative assessment would be required where a structure showed signs of distress.

# 5 Analytical analysis

## 5.1 Background

According to Kerisel (1992), from the end of the 10th millennium BC mankind has been building earth-retaining structures such as ramparts as protection against intruders. Whilst their builders would not have known how to calculate the earth pressures involved they were clearly capable of proportioning their structures to sustain them (Kerisel, 1985). Given that the purpose of these early retaining structures was predominantly defensive, it is perhaps not surprising that military engineers were at the forefront of developments in earth pressure theory. The essay by Coulomb in 1776 marked the culmination of these efforts (Heyman, 1972) and with it the birth of soil mechanics as we now know it.

In Coulomb's method of calculating earth pressures on retaining walls, the force imposed on the back of a retaining wall by a rigid wedge of soil sliding on a plane shear surface, together with the cohesive and frictional forces acting to resist movement along that surface are considered. Coulomb recognised that there could be frictional as well as normal forces acting on the back of a retaining wall and also that the effects of water could both increase the pressure on the wall and weaken the soil behind it. A detailed explanation of Coulomb's theory is provided in Clayton *et al*. (1993).

Ensuing developments of earth pressure theory have been traced to modern times by Skempton (1979 and 1985) and by Peck (1985).

## 5.2 The need for analytical methods and their validation

The emphasis in the literature is on analytical methods, but back-calculation from experience in the field and from scale models to verify theory is also covered. For example, the paper by Baker (1881) presents field and experimental evidence on the magnitude of earth pressures, much of it collected during the building of the Metropolitan and District Underground Railway Lines in London. In attempting to show that the earth pressure theories of that time overestimated the lateral thrust and overturning moments, Baker overlooked the role of wall friction. More perceptively, he deplored the lack of experimental data on the magnitude of earth pressures and reproached client authorities for not undertaking such studies. Following the publication of that paper, numerous field observations and experiments have been carried out to determine the pressures exerted by the ground on retaining structures so that today there is a much better understanding of the mechanisms involved. However the problem is not straightforward as the ground pressures, acting at any given instance, depend on minor variations in the deformation of the retaining structure and the ground behind it, the way these interact and the pore pressure regime in the soil. The difficulties inherent in modelling such factors in a numerical method of analysis does not mean that such methods have no role to play in furthering the understanding of the behaviour of masonry-faced walls, or in developing more closely-defined assessment criteria for them. The latter should reduce the range of inferences drawn by different inspectors/assessors from identical situations.

Any method of analysis needs to be validated against actual behaviour. Large-scale experimental civil engineering works are expensive, particularly where extensive instrumentation is required and where long-term performance is under investigation. Furthermore, with such works it is rarely possible to investigate, in a controlled or systematic manner, the effect of changing the value of the governing variables. For many situations, centrifuge modelling offers a convenient means of completing such parametric studies: numerous references can be found in the literature. However, emphasis has been given, increasingly so, to the use of numerical methods for parametric studies. This trend away from physical testing to a numerical approach has been encouraged by the relatively low level of investment in site monitoring works in the UK. For this study, it was fortuitous that full-scale trials had been carried out on four dry-stone retaining walls at Kingstown, now Dun Laoghaire, in Ireland in

1834 (Burgoyne, 1853): it is pertinent to note that no other full-scale experiments on dry-stone walls seem to have been undertaken since. (Brief details of the analyses undertaken at Kwun Lung Lau and at Great Zimbabwe are given at the end of Section 5.3.4.)

## 5.3 Methods

There are a number of analytical methods available to designers and assessors of in-service structures. They can be categorised as follows.

### *5.3.1 Closed-form solutions*

Closed-form solutions provide a useful means of checking the results of other methods but, because such solutions are based on a number of quite limiting assumptions, rarely are the results of direct use for practical applications. For example, the materials making up the wall and the backfill might be assumed to be homogenous and linearly elastic, which is hardly likely to be the case for a masonry-faced retaining wall.

### *5.3.2 Limit equilibrium method*

At the risk of over-simplification, in this method a slip surface through the domain is assumed and analysis proceeds by the application of relatively simple static equilibrium equations to determine the factor of safety (or some other measure of safety) along the surface. For example, the slip circle method quantifies the factor of safety against sliding along an assumed failure surface, and from a series of analyses a minimum factor of safety can be derived. Coulomb's method for analysing the stability of a retaining wall follows much the same approach to derive the limiting forces acting on the back of the wall. Because a search will not necessarily identify the most critical failure plane, such an approach will, in general, overestimate the factor of safety: thus it is termed an upper-bound approach.

### *5.3.3 Stress field method*

Simply stated, in this method a stress field is constructed within the domain such that at no point is the failure criterion for the material(s) contravened; that is, the domain is in equilibrium with the external disturbing forces. Because of the inefficiencies of constructing such a field, such an approach will, in general, underestimate the factor of safety: thus it is termed a lower-bound approach.

(Note that the combination of a limit equilibrium method and stress field method can bound the actual factor of safety: in some rare cases they might give the same 'exact' solution.)

### *5.3.4 Numerical methods*

With a numerical modelling technique, the domain is divided into numerous elements, and analysis proceeds through consideration of the interaction of these and the conditions at the boundary of the domain. There are four basic approaches: (i) finite element analysis (FEA), (ii) boundary element analysis, (iii) finite difference methods, and (iv) discrete element analysis (DEA). Harris (1992), for example, provides details of the numerical methods used to analyse reinforced soil structures.

Numerical methods can do all and more than the three other methods. Whilst all the methods can be used to assess the collapse limit states of a structure, a numerical analysis can also assess the likely deformation of a structure during its construction and in response to in-service loads. That said, predicting the deformation of some structures is notoriously difficult; for example, predictions of the lateral deformation beneath an embankment constructed on soft ground are usually wide of the mark. (The error is partly due to the difficulty of characterising the stiffness of soils at low strains.)

In most non-numerical methods, a retaining wall will be considered as a monolith: that is, it would retain its shape and integrity up to collapse. This is certainly not the case for dry-stone walls and their derivatives that are composed of a multitude of individual elements, which are at best only poorly bonded to each other. These assemblages are incapable of resisting tensile stresses of any significant magnitude and can tumble down during collapse. This simplifying, non-conservative assumption of coherence need not be adopted in a numerical method of analysis. Thus numerical methods seem better equipped for assessing the stability of in-service dry-stone walls but their application is limited because of the relatively high cost of undertaking an analysis: at present, such an analysis would be unwarranted for most earth retaining structures.

*Finite element analysis*

With FEA, the domain is modelled as a continuum with internal forces balancing the externally applied loads.

Finite element analysis was developed during the Second World War in response to the need to analyse the behaviour of airframes, and for which there were no other suitable methods available other than the empirical 'try-it-through-fly-it' approach. Successful practical application of FEA required the development of computers. There is a substantial body of information in the literature on the theory and application of FEA to ground engineering problems.

Analysis of ground engineering problems can use a range of soil models ranging from linear elastic, to non-linear elastic-plastic critical state formulations. Brady *et al.* (2000) reviewed the use of FEA at TRL for an earlier internal reinvestment project: over the years success has been achieved using FEA to analyse soil-structure interaction problems - notably by Higgins *et al.* (1989) for reinforced concrete embedded retaining walls. However the behaviour of soils is complex and in many cases cannot be modelled accurately with simple models, particularly soils that dilate during shearing. Furthermore there might be problems in using a FEA package to analyse the collapse of a soil structure - as might be anticipated these mainly concern the validity of the assumption that the domain acts as a continuum. It is difficult to model situations where joints, cracks, slip surfaces and the like occur in the domain: such discontinuities affect, for example, the distribution of stresses within the domain and its permeability (perhaps by orders of magnitude). And it is particularly difficult to model the growth of a network of bifurcating cracks - it being necessary to divide elements into ever smaller sub-elements. (The time to complete an analysis increases with the number of elements.) FEA packages can be used readily and successfully to investigate problems where small strains or movements occur, but the application of FEA (or indeed any numerical method) for complex situations is not for a casual user.

*Boundary element analysis*

In a boundary element analysis only the surface of the domain is divided into elements. This technique has been used to model the interaction of bodies.

*Finite difference analysis*

Finite difference analysis is similar to FEA in that the domain is divided into numerous elements. However, whereas FEA derives a continuous solution, finite difference calculates a solution at specific points in the domain. This has some advantages where the differential equations are complex, or indeed insoluble, but a major disadvantage is the difficulty of using the method to model complex geometric problems.

*Discrete element analysis*

With discrete element analysis (DEA), the domain is divided into a number of interacting blocks and the interaction between them is defined through, for example, an elastic/plastic model.

Discrete element codes were developed in the 1970s to investigate the performance of jointed rock masses, the best known in the UK is UDEC (Universal Distinct Element Code), Itasca (1993), which was developed from the work of Cundall (1971) and Cundall and Strack (1979). With such codes, a problem is solved dynamically with block velocities, rotations and positions recalculated repeatedly after small increments of time. In other words they deal with pseudo-static problems by allowing the dynamic behaviour to reach equilibrium (or very nearly so) or the final state in a notional time. Equilibrium can be judged by the magnitude of the ratio of the out-of-balance and *in situ* forces. The time to reach a solution is a function of the number of elements and the number of increments, and the approach can be very demanding in terms of computer power - particularly where collapse conditions are investigated. This demand restricts the application of such codes; for example, at present, only rarely will a 3-D analysis be undertaken but, given the inexorable growth in computer power, this limitation will reduce with time.

The interaction between hard rotund particles can be described in terms of an elastic modulus and a coefficient of friction, and so the development of a slip surface through a collection of such particles can be predicted quite successfully. DEA has been used to model the behaviour of fractured materials: the fractures might have been there from the outset or they might have been generated during the analysis. However it is difficult to model particle crushing and only then at the expense of increasing computer run times. It is important to appreciate that apart from research work, principally undertaken in powder technology, individual particles are not modelled in a DEA. In most cases, a model is used to describe the behaviour of each of the materials in the domain - usually the models are the same as used in an FEA. And, as with FEA, there are difficulties in accurately modelling the volume change (dilation or compression) that occurs through shearing. Without modelling the individual grains it is difficult to accurately predict the behaviour of a dense mass of angular particles or a collection of particles of widely differing shapes and sizes. Such considerations are the subject of research and are outside the scope of this investigation, but it should be appreciated that DEA is not a panacea for analysing the behaviour of soils.

*Application of DEA to analysis of behaviour of masonry-faced earth retaining walls*

In the investigation of the collapse at Kwun Lung Lau, previously mentioned, some 22 analyses using UDEC were undertaken. In one particular analysis the wall bulged initially; bulging continued until brittle fracture of the wall occurred at about mid-height with overturning of the wall below this level. It also indicated that the upper part of the masonry wall would rotate backwards as a result of the displacement of the sliding mass of soil and predicted that it would come to rest essentially intact on the surface of the debris; and this is what had happened.

Analyses using UDEC were undertaken by Dickens *et al.* (1993) on the free-standing dry-stone walls up to 10 m high at Great Zimbabwe. The outputs from this numerical modelling were successfully compared with physical tests in the laboratory on 2 m high, 1 m wide and 3 m long dry-stone walls constructed of blocks of rock quarried on site and built by a stonemason from Zimbabwe.

**5.4 Defining safety**

Most analyses are undertaken to quantify the level of safety: this might be expressed in a number of ways - for example, as a factor of safety, a mobilized strength, or in statistical (probability/reliability) terms. A discussion of what constitutes an acceptable minimum level is outside the scope of this report but, inevitably, it should be firmly anchored to past experience and current knowledge. However expressed, the level of safety built into a design is intended to more than cover the gap between the actual and calculated in-service performance: it is a measure of the level of uncertainty.

An assessment method must review the possibility of occurrence of the various failure modes whilst taking account of the age of the structure. What is required, therefore, is an assessment system where the level of safety (however expressed) reduces (is traded-off) in line with the age of a structure in recognition of the ‘knowledge’ gained over time on its performance. This kind of approach might well be adopted when deriving an insurance premium for an ancient monument, such as the Pantheon, but it is not widely used in civil engineering practice. (According to Heyman (1988) the dome of the Pantheon has a geometrical factor of safety of about 1.7, which is below the level required by current codes.) Indeed when assessing the stability of a dry-stone wall, an advanced age is more likely to count against it rather than for it.

### *5.4.1 Overturning*

The factor of safety against overturning is conventionally derived as the ratio of the restoring and disturbing moments. Although the forces acting on the body are in equilibrium the equation represents an inequality (restoring > disturbing moments) other than for a factor of safety of unity. Other factors of safety might be derived in terms of:

1 the ratio (b/2e) where (e) is the eccentricity of the resultant force acting at the base of a wall of width b: overturning does not occur until e > b/2;

2 the maximum pressure developed at the toe of the wall.

The latter would not be of much use where a wall was built on a rigid (or nearly so) foundation, as were the walls at Kingstown. However in most cases it would provide the best measure of serviceability for it is this in combination with the compressibility of the foundation that dictates the differential settlement through a wall (that is, from toe to heel). Differential settlement might induce an outward lean of a wall face, and this would increase the overturning moment and, in turn, the foundation pressure: in the extreme this progressive process will lead to the collapse of a wall by toppling.

Considering the stability against overturning of a monolithic wall of height (h), width (b) and weight (W), as shown in Figure 1, where the active earth pressure ($P_a$) acts at an angle $\phi$ to the horizontal on the back of the wall such that $\phi = \tan^{-1}(h/3b)$. For this value of $\phi$, $P_a$ passes through A at the toe of the wall, and $P_h = P_a \cos\phi$ and $P_v = P_a \sin\phi$.

*Factor of Safety (FS)*

Case 1. Using $P_a$ and taking moments about A

$$FS = \frac{Wb/2}{0} = \infty$$

Case 2. Using $P_h$ and $P_v$ and again taking moments about A

$$FS = \frac{Wb/2 + bP_v}{P_h h/3}$$

$$FS = \frac{Wb/2 + bP_a \sin\phi}{bP_a \cos\phi \tan\phi} = \frac{Wb/2 + bP_a \sin\phi}{bP_a \sin\phi}$$

$$FS = \frac{Wb/2}{bP_a \sin\phi} + 1 \neq \infty$$

Case 2 might be preferred to Case 1 on the basis that a factor of safety of infinity can be derived for a situation that apparently has a finite value. But Case 1 shows that the situation in Figure 1 represents a limiting point. With the force $P_a$ acting at h/3, for a slightly higher value of $\phi$ the trajectory of the earth pressure force would pass through the foundation showing that outward overturning would not occur about the toe, whilst a slightly lower value of $\varphi$ would give a high factor of safety against overturning. This differentiation is not apparent with Case 2: with this, a value of infinity for the factor of safety is only obtained with $P_a = 0$ ($\phi > 0°$).

Furthermore, for the general case, a change in the density of the backfill or wall blocks has a proportionate effect on the value of the factor of safety derived for Case 1 but not Case 2. For example, doubling the density of the backfill or halving the density of the wall blocks would halve the factor of safety determined using Case 1 but not with Case 2. Thus with Case 2, a factor of safety of 2 does not mean that half the available overturning capacity is mobilised – perhaps a particularly telling objection to its use.

*Margin of safety (MS)*

Case 1

$$MS = Wb/2 - 0 = Wb/2$$

Case 2

$$MS = Wb/2 + bP_v - hP_h/3$$

$$MS = Wb/2 + bP_a \sin\phi - bP_a \cos\phi \tan\phi = Wb/2$$

Thus, as could be anticipated, the margin of safety is independent of how the forces are treated. But the level of safety is a function of the margin relative to the disturbing (or restoring) force, as so the question of how this should be calculated remains. Note that for Case 1, the disturbing force (DF) is zero and so MS/DF = ∞.

Assigning values to the variables:

Let b = 1, h = 1.5, and so $\phi = 26.6°$, and take $\gamma$ (wall and soil) = 20 kN/m²

Then W = 30 kN, $P_a$ = 8.58 kN and so

FS (1) = (15/0) = ∞

FS (2) = (15/3.84) + 1 = 4.9.

Margin of safety = 15

*Geometric factor of safety (b/2e)*

$R_v$ = 30 + 3.84 = 33.84

And so, taking moments about A, x = 15/33.84 = 0.443, and b/2e = 8.8.

This confirms that the wall has a high, but finite, factor of safety against overturning. Note that for e = 0 (so that b/2e = ∞) the lines of action of $P_a$ and W must cut the base at the same point: this is a different physical limit to that shown in Figure 1, for this $\phi = 45°$.

The above analysis serves to highlight the anomalies in the factor of safety approach in the assessment of wall stability. The conventional factor of safety for overturning does not give a unique solution: its value depends on whether the resultant earth pressure acting on the back of the wall is treated as a vector or as resolved components.

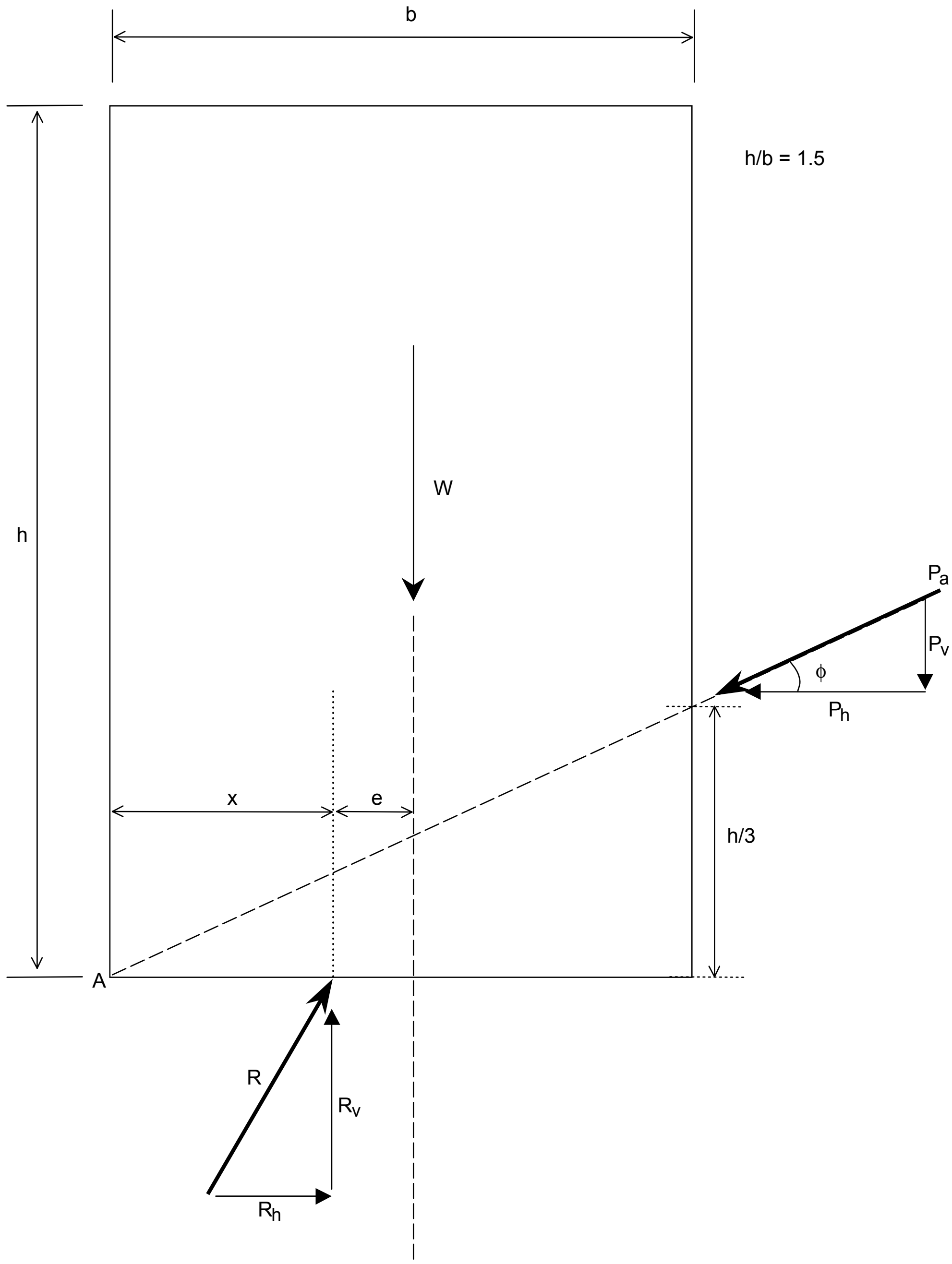



**Figure 1** Overturning of block wall

This is unfortunate: it means that the intuitive feel engineers have, or think they have, for a desirable value for this factor might be misleading. Note that this problem of defining a factor of safety against overturning is not confined to masonry-faced earth retaining walls.

The data in Figure 2 are in a more practical range. These derive from calculations undertaken for Wall C at Kingstown. In these calculations the wall is treated as a monolith and built to its full height of 6.1 m prior to backfill being placed in layers behind it. The figure shows how the factor of safety calculated using (a) $P_a$, (b) its components $P_n$ and $P_p$ (which act normal and parallel to the back of the wall respectively), and (c) b/2e (the geometric factor of safety), varies with the height of backfill. For the parameters used, the wall remains stable until the backfill reaches a height of about 5.8 m and at this point both methods of calculation give an identical factor of safety of unity. (This is some 0.6 m or so higher than achieved in practice, the difference stems from the treatment of the wall as a monolith.) Further discussion on the factor of safety against overturning is given in Section 10.1.

### *5.4.2 Current requirements*

The current Code of Practice for Earth retaining walls (BS 8002: 1994) states that a gravity structure should be checked against overturning, but it provides no direction on how this should be done or what level of safety is required. It does, however, provide guidance on checking the bearing capacity of a gravity wall. Its predecessor, CP2 (1951), states that for a gravity wall the resultant thrust

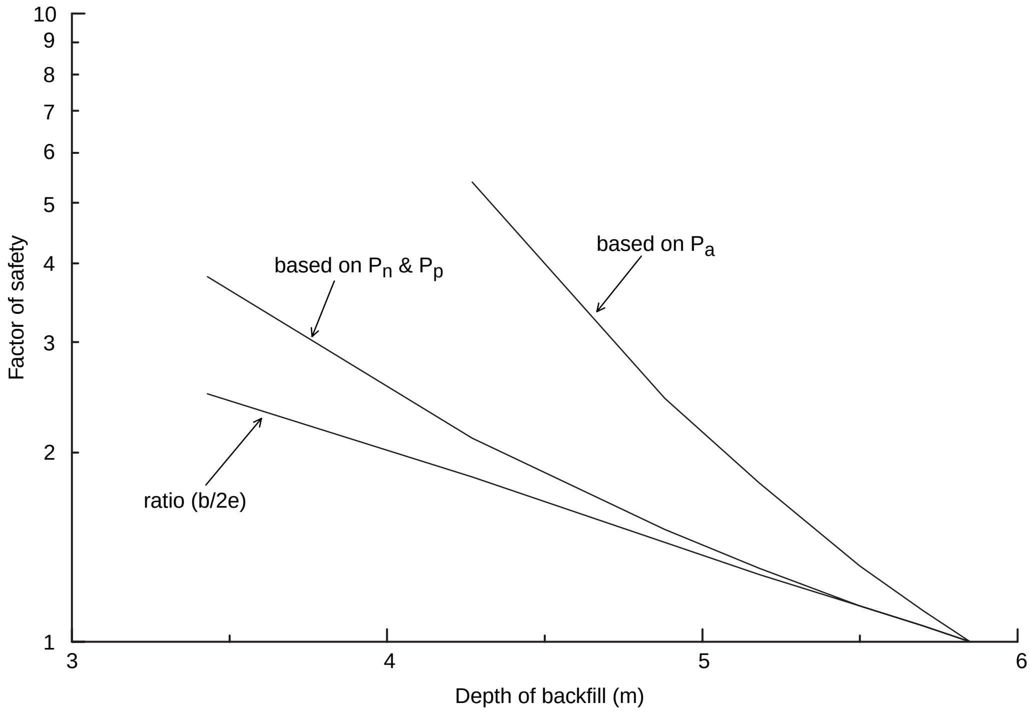



**Figure 2** Variation of factor of safety with depth of backfill, based on calculations for Wall C at Kingstown

should fall within the middle third of the base; as discussed in Section 10.1 this seems overly conservative and does not ensure that a tension crack is not generated within a dry-stone wall. Furthermore, whilst the document states that a minimum factor of safety of 2 against overturning is required for other types of retaining walls, it does not say how this factor should be calculated. The text given in both these documents suggests that gravity retaining walls are suitable for walls up to 3 m or so in height: much higher masonry-faced walls are found on the highway network across Europe.

By their nature, design codes inevitably concentrate on the structural forms in vogue at their time of issue. This presents a problem for inspectors and assessors as they are often required to deal with supposedly outdated structural forms. This goes some way to explain why BS 8002: 1994 is hardly applicable to masonry-faced walls despite the fact that they account for about 80 per cent of all retaining walls on the highway network in Great Britain. Similarly, the emphasis given to checking the bearing capacity of the foundations to a retaining wall reflects the fact that most new walls will be founded on compressible ground. But dry-stone walls are usually constructed of the local ‘country’ rock and so many up-slope walls will be founded on the floor of a rock excavation, see Jones (1979) and Gutpa and Lohani (1982).

The limited applicability of current design standards for assessment purposes is not confined to masonry-faced walls. This supports the view that assessment standards are essential to ensure the proper, economic maintenance of the current stock of structures. It will be appreciated that in the UK, for example, the value of the new stock of highway structures built each year represents a small proportion, say about 1 per cent, of the existing stock. It would seem that the role of a maintenance engineer is perhaps undervalued relative to that of a designer: often the former has to deal with problems bequeathed by the latter.

## 6 Details of the experimental walls at Kingstown

As with any gravity retaining structure that depends upon its weight to resist the disturbing earth pressures acting upon it, the shape of a block earth retaining wall has a significant bearing upon its stability. To investigate the effect of geometry on stability, Burgoyne (1853) constructed four dry-stone retaining walls having the same volume of stone per unit length but different cross-sections. Schematic views of the walls are provided in Figure 3.

Wall A had a uniform thickness of 1.02 m (approximately one-sixth of its height) and was battered back at a slope of 1 in 5.

Wall B varied in thickness from 1.63 m at the base to 0.41 m at the top and had a vertical back.

Wall C was of the same dimensions as Wall B but it had a vertical front face.

Wall D was a plain vertical wall with a uniform thickness of 1.02 m.

Each of the walls was constructed from blocks of roughly squared granite, laid dry; i.e. without mortar. The walls were backfilled with loose earth having an initial placement density of about 1390 kg/m$^3$, although the density of the fill would have been increased by rainfall during construction operations and trafficking by construction workers tipping earth from wheel barrows.

In each case, the masonry facing was built up as the backfill was placed. Wall A reached its full height of about

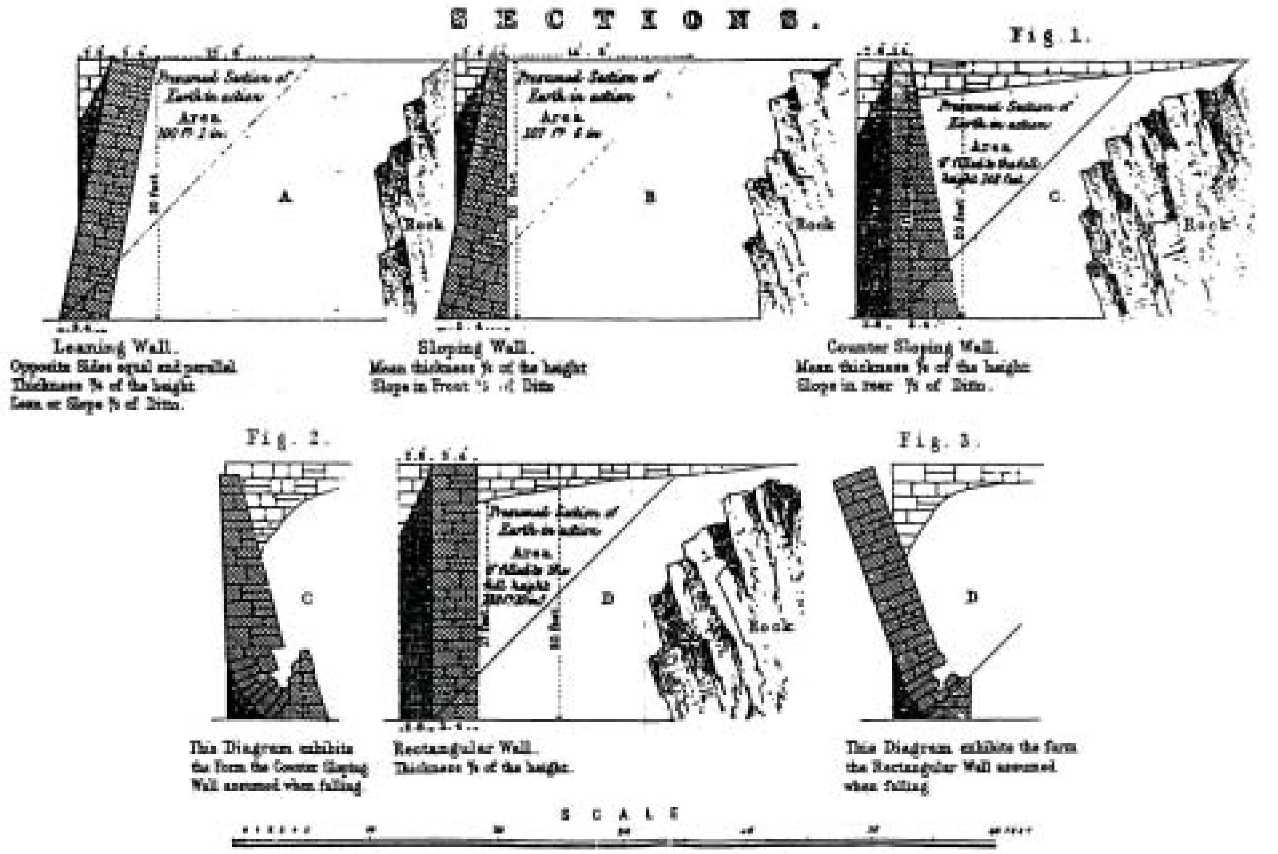



**Figure 3** Cross-sections and failure mechanisms of walls tested by General J Burgoyne in 1834 (from Burgoyne, 1853)

6.1 m (20 ft) with no sign of distress. Wall B also reached its full height but exhibited an outward movement of 63 mm at the top of the wall, together with some fissuring in the masonry face. Both walls C and D collapsed when the backfill reached a height of about 5.2 m.

## 7 Details of UDEC analyses of walls at Kingstown

In July 1996, the TRL commissioned the Geomechanics Research Group, Department of Civil and Environmental Engineering, University of Southampton to carry out a series of DEA analyses of the full-scale tests at Kingstown. Further analyses were commissioned in 1998. The second author, with the invaluable assistance of Mr X Zhang, completed the final series of analyses using the facilities at the University.

### 7.1 Boundaries

All the analyses described in this Report were carried out in plane strain, using the cross-sections shown in Figure 4. In each analysis, the bottom of the mesh was pinned, thereby preventing movement in both the horizontal and vertical directions, and the right-hand vertical boundary was prevented from moving vertically.

### 7.2 Mesh

By way of example, the mesh used for wall B is shown in Figure 5. In some of the early analyses, particularly small elements were used to determine the stress distribution at the toe of the wall so that the conditions at the onset of collapse were well defined. This was not so important in the analyses undertaken to examine the effect of joint pattern on the performance of the walls and so, to reduce run times, such a fine mesh was not used in these runs.

The block patterns used in the analyses were as given by Burgoyne, and as reproduced in Figure 3, but these would have been merely schematic. The effect of the pattern was investigated in the latest series of analyses.

### 7.3 Construction

For wall A, the construction of the wall and placement of the backfill proceeded by adding wall elements and soil elements simultaneously to give a lift of 0.6096 m (2 foot): as was the case in practice. The other walls were constructed in two stages and in advance of placing the backfill: the backfill was again placed in 0.6096 m lifts, except for the failing walls (C and D) where the final layer was only 0.3048 m thick.

### 7.4 Material models

An elastic/Mohr-Coulomb plastic model was chosen for the masonry blocks, the backfill and the natural rock. Such a model was also used to simulate the behaviour of the interfaces between the blocks.

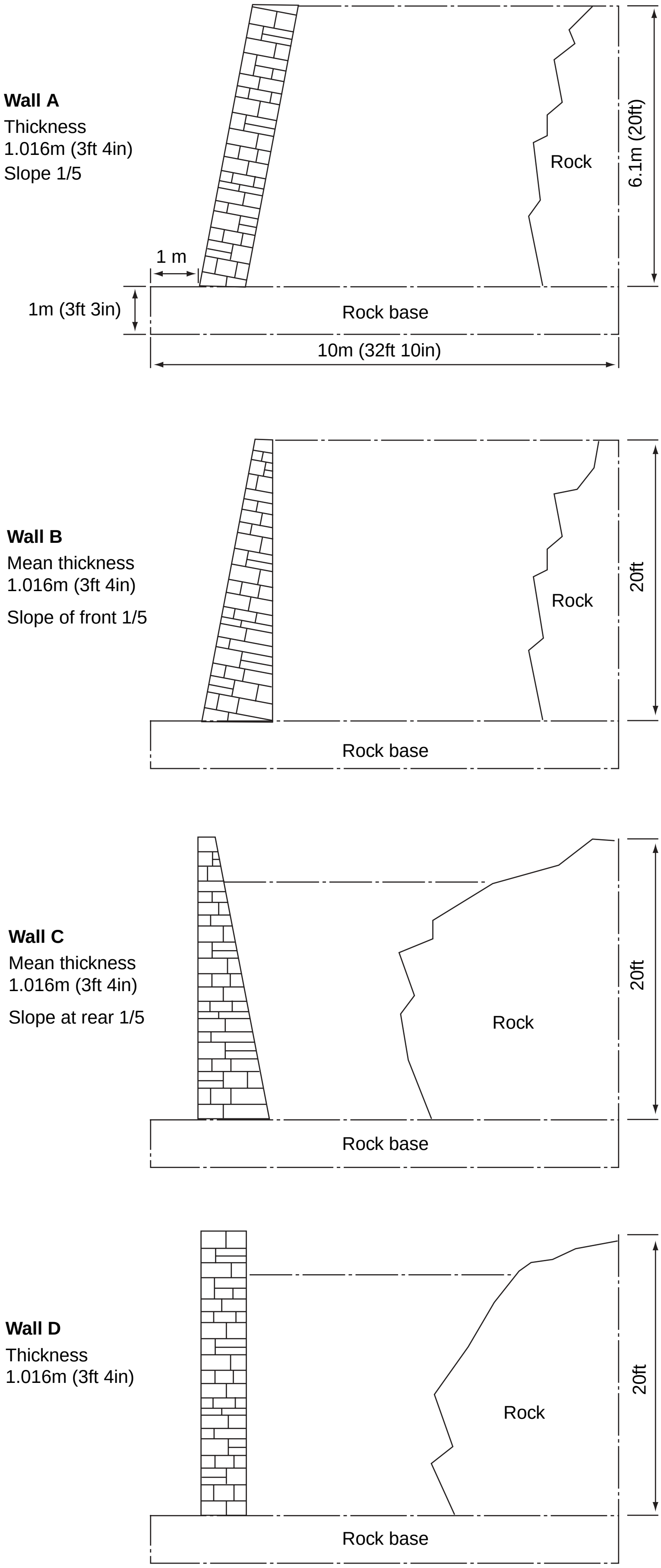



**Figure 4** Cross-sections of walls modelled using UDEC

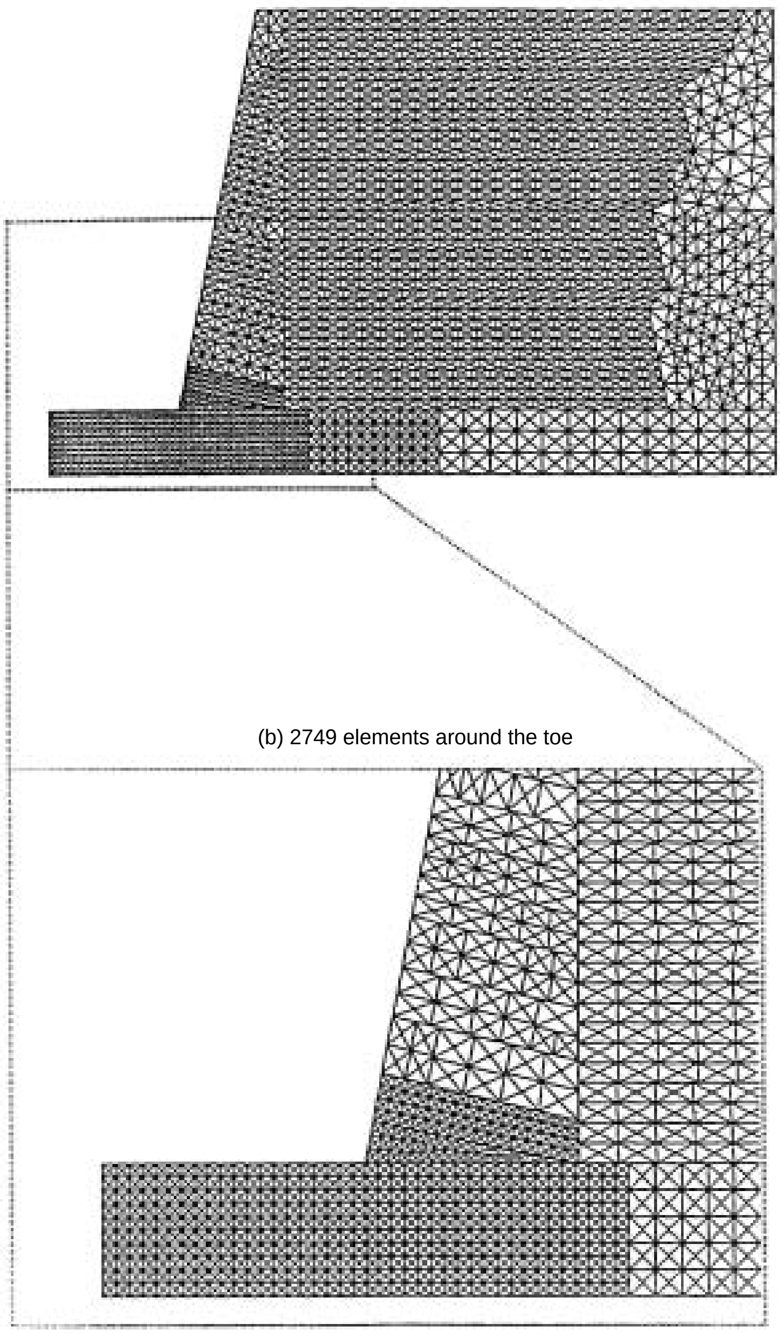



**Figure 5** Typical finite element subdivision of discrete blocks (Wall B with detail around toe), after Harkness *et al.* (2000)

## 7.5 Material properties

### *7.5.1 Facing blocks and bedrock*

Both the masonry blocks and the rock outcrop were granite: it seems probable that the blocks were obtained from a quarry located near to the site. From published data, the bulk modulus of the outcrop was taken to be 22 $GN/m^2$ and the shear modulus was taken as 1.5 $GN/m^2$.

According to Burgoyne, the density of the granite blocks was 2270 $kg/m^3$. This is about 14 per cent lower than quoted for intact granite (about 2650 $kg/m^3$), by Goodman (1980). The difference corresponds to a gap width of about 5 per cent of the thickness of the blocks, which seems reasonable for roughly squared blocks as were used in the wall.

There is little published information from which to derive values for the normal stiffness and shear stiffness of the interface between the granite blocks. The stiffness of an interface is a function of the surface topography and properties of the bulk material. Burgoyne stated that the granite blocks were roughly squared and so a relatively low normal stiffness of 1 $GN/m^2$ per metre thickness was adopted in the analyses. Similarly, the shear stiffness of the interface was taken to be 0.5 $GN/m^2$ per metre. These parameters affect the deformation of the blocks and movements between them and so have some influence on the outward deformation of a wall. The effect of varying these parameters was not investigated in this study because they were unlikely to much affect the stability of the walls at Kingstown.

Based on the data provided by Goodman (1980), the angle of friction between the granite blocks was taken to be 45°. However the effect of varying this value was investigated in some of the later analyses.

To avoid numerical instability it was necessary to describe the roundness of the corners of the blocks. In the initial analyses a corner radius of 10 mm was adopted, but the effect of the radius on performance was investigated in later analyses.

#### *7.5.2 Backfill*

According to Burgoyne the backfill was '*thrown in loose and placed without any precaution of rammimg or otherwise'*. He gives the mass density of the backfill, '*as put in*', as 1390 kg/m$^3$ (87 lb/ft$^3$), but also states that it was necessary to make good '*the deficiencies from subsidence in the filling*'. Baker (1881) suggests that the density would have increased as the soil '*imbibed the rain and the moisture*' to at least 1796 kg/m$^3$ (112 lb/ft$^3$). However, ignoring compaction effects, this density would have required an infiltration equivalent to about 2500 mm of rainfall and so it seems implausibly high: it is about four times the annual precipitation for Dublin. Some densification of the backfill would have occurred as a result of trafficking and self-weight effects: Burgoyne's records suggest that settlements of up to 200 mm occurred, which is equivalent to 3 or 4 per cent of the height of the walls. Taking all the above into account, the mass density of the backfill probably fell within the range 1450 to 1550 kg/m$^3$. In determining the effect of density on stability, the first series of analysis was undertaken using density values of 1400 kg/m$^3$, 1550 kg/m$^3$ and 1700 kg/m$^3$.

Burgoyne did not provide any details of the stiffness of the backfill: this is hardly surprising given that soil tests were not standardized until the 20$^{th}$ century. However, based on the properties of similar soils, the bulk modulus of the soil was taken to be 1 MN/m$^2$ at the surface and increasing at a rate of 0.5 MN/m$^2$ per layer of soil (about 0.3 m thick) with depth. Similarly, the shear modulus of the soil was taken to be 0.6 MN/m$^2$ at the surface and increasing at a rate of 0.3 MN/m$^2$ per layer of soil.

From the description of the soil provided by Burgoyne, the angle of friction of the backfill was estimated to be 28°. However a range of values was used in the first series of analyses.

During construction the soil would settle relative to the rough granite blocks, thereby generating a downward acting interface frictional force. Because the blocks were rough, it was assumed that full friction could be generated on the back of the wall and so the maximum angle of wall friction was assumed to be equal to the angle of friction of the soil.

#### *7.5.3 Run times*

The rate of iteration varied with the details of the mesh and also, at a particular stage of construction, with the stability of the wall at that stage. For most of the analyses covered in this report, the rate varied from about 100k cycles per hour during the early stages of construction to about 250k cycles per hour as collapse was approached. Some of the analyses reported by Harkness *et al.* (2000) required about four weeks (about 700 hours) of continuous processing to complete. For economy and practicality, the number of cycles for some of the later analyses was limited. For example, in investigating the effect of the joint pattern on performance, a standard routine comprising 1.9 Million cycles was initially adopted: such an analysis took between 8 and 15 hours to complete. Further cycling was undertaken for a few runs where collapse was approached: these continued up to a maximum of 2.5 Million cycles and commonly took 19 hours or so to complete: even so equilibrium conditions or collapse was not always achieved. The run times could have been much reduced by reducing the number of elements in the wall and backfill, and also by using more recent hardware.

## 8 Results of UDEC analyses of walls at Kingstown

As mentioned earlier, Walls C and D collapsed when filling reached a height of about 5.2 m, whilst Walls A and B were stable at their full height of 6.1 m. Using the best estimates for the input variables, as described above, the UDEC analyses reproduced this behaviour. Moreover the predicted deformation patterns close to failure of Walls C and D reproduced the actual behaviour where a triangular pile of stone blocks remained behind the toe of the wall following collapse.

It should be appreciated that obtaining such a good agreement does not completely validate the method of analysis nor the input values chosen for the variables. Given the number of variables that affect stability, it is possible that this agreement was the result of compensating errors in the values of some of the input variables. Furthermore the predicted movements of the stable walls did not agree particularly well with the measurements reported by Burgoyne: there are a number of possible reasons for the discrepancy. Nonetheless the fundamental properties of the backfill seemed to be reasonably well bounded by the agreement. With this in mind, the later analyses using UDEC were mainly aimed at determining the effect of certain variables on the stability of the collapsed walls, and on the deformation of the intact walls.

It would be impractical, as it is unnecessary, to reproduce the results of all the UDEC analyses. The following presents selected conclusions and observations from the results of the UDEC analyses as reported by Brady and O'Reilly (1999), Harkness *et al.* (2000) and latterly by Brady and Kavanagh (2001).

The generation of horizontal displacement during the construction of the walls is shown in Figure 6. As shown there, the predicted displacements of Walls A and B at the end of construction are 12.8 mm and 32.6 mm respectively. According to Burgoyne, on completion of construction the outward deflection of the top of Wall B

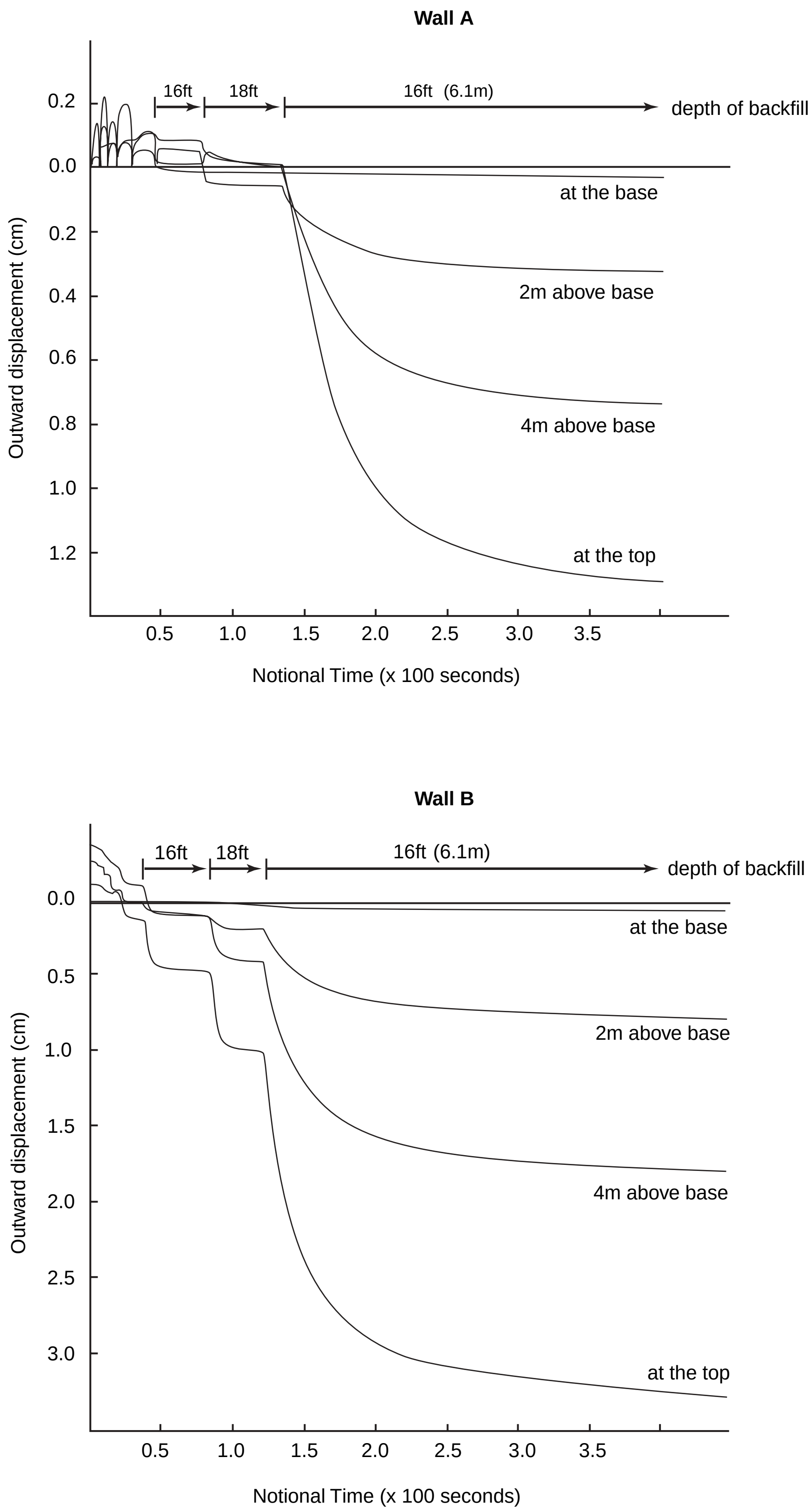
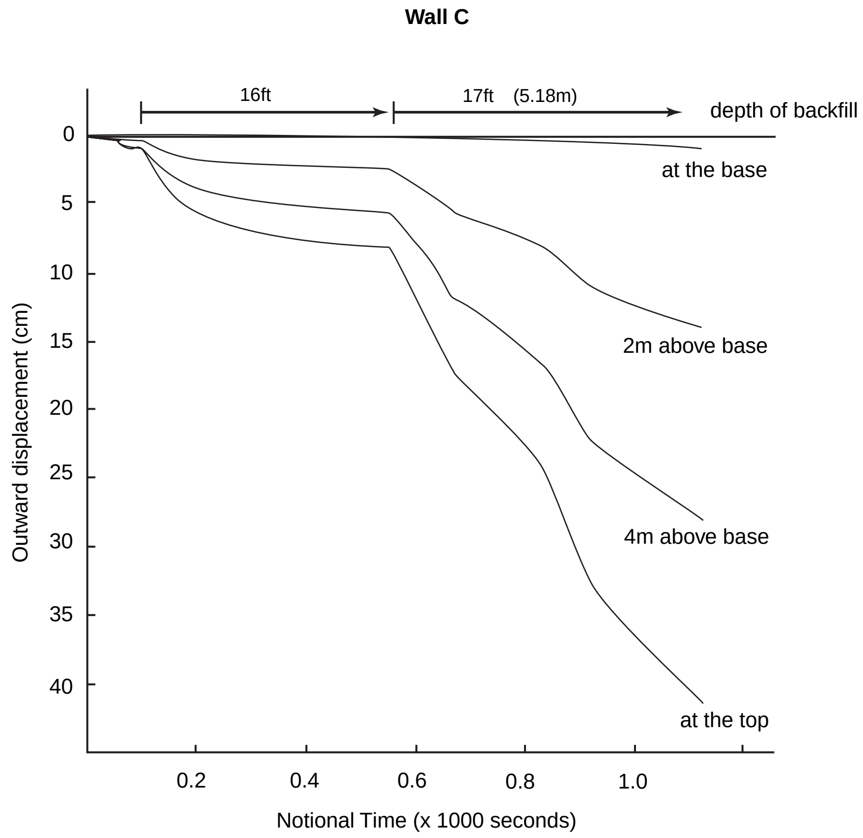



**Figure 6a** Horizontal displacement of walls during placement of backfill, after Harkness *et al*. (2000)



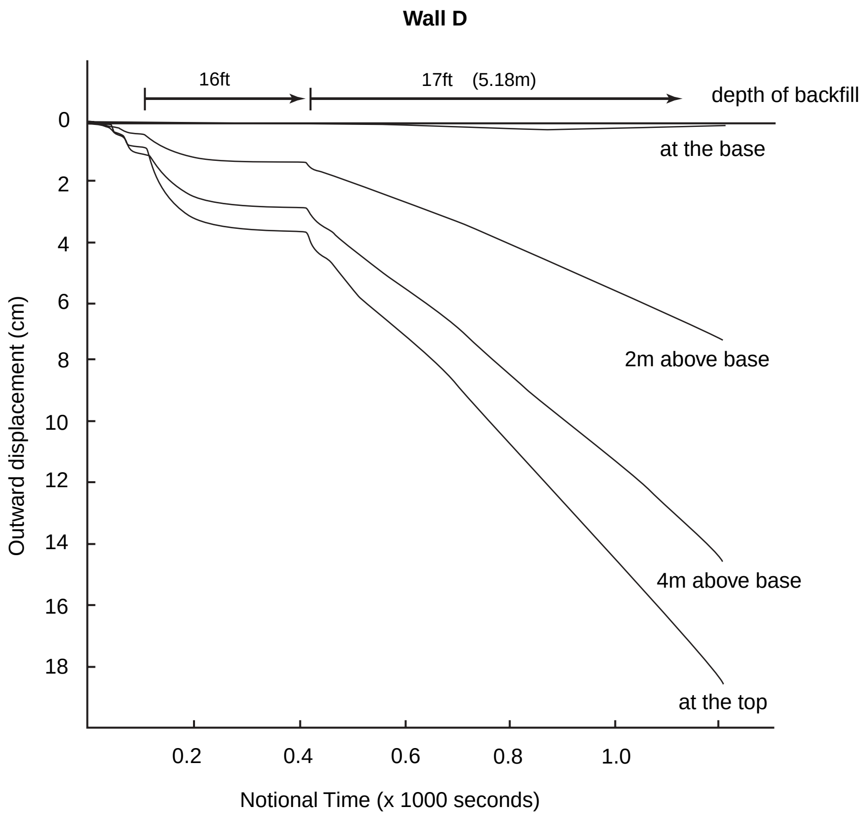




**Figure 6b** Horizontal displacement of walls during placement of backfill, after Harkness *et al*. (2000)

was 63 mm (2.5 inches), but there was nothing to suggest that the wall was then in a critical condition although there were '*some slight fissures in the face*' which were considered to be '*indicative of instability*'. The results of the analyses show that Wall A was the most stable again in agreement with actual behaviour: according to Burgoyne that wall '*remained unaffected by the lateral pressure of the earth filling*' when filled to its full height. The results of the analyses indicate that the assumed shear stiffness of the blocks was higher than the actual value.

As shown in Figure 6b, the results of the analyses show that the deflection of the top of Walls C and D increased quite rapidly as failure was approached. This might suggest that there would be little warning of a failure of an in-service structure. The results of a number of analyses taken to collapse suggest that the top of these dry-stone walls could deflect up to about 100 mm or so (or 1/60$^{th}$ of their height) before the onset of collapse. However the maximum allowable movements might depend upon the geometry of the wall, and movements can be generated by mechanisms other than by overturning about the toe.

The predicted distributions of horizontal stress acting on the walls are shown in Figure 7. Also shown are those calculated using active earth pressure coefficients as interpolated from the tables provided by Caquot and Kerisel (1948). The good agreement strengthens the view that the input values for the density and strength of the backfill are reasonably accurate, or at least that the combination of values is. It would commonly be assumed that active pressures would only be generated on the back of a wall that was close to collapse, but a better interpretation is that the outward deflection of these walls was sufficient to generate such pressures. The backfill was placed in a relatively loose condition and so the movement to attain active conditions from the initial *in situ* conditions might have been small.

Back-analysis of the distributions of stress also shows that full interface friction was generated on the back of the walls.

The predicted normal stress distributions on the base of the walls are shown in Figure 8: also shown are the distributions derived from consideration of limit equilibrium of the forces due to the backfill and to the weight of the wall face (assuming the walls act as monoliths). The data from the analyses show that:

1 For Wall A, the contact pressure was negligible over about 20 per cent of its base width, and so the ratio of (b/2e) was about 2.

2 For Wall B, the contact pressure was negligible over about half the base width, and so the ratio of (b/2e) is about 1.6.

3 For Walls C and D, the resultant force acts through the toe of the wall.

On this basis Wall A is more stable than Wall B, and Walls C and D are unstable - as stated already the latter pair collapsed.

The possibility of sliding failure along the base of a wall has not been investigated in any particular detail in this study, but it is recognised that this mode of failure might be critical, particularly so for walls built on weak foundations.

### 8.1 Effect of wall variables on performance

#### *8.1.1 Interface friction angle between blocks*

As described earlier, on the basis of published information the angle of friction between the blocks was taken to be 45°. However analyses were undertaken using lower angles to explore the transition between wall failure by overturning of the wall and by sliding through the wall. The results of some analyses are summarised in Figure 9. For some data points shown here, equilibrium conditions were only achieved after some outward movement of the wall, and accompanying settlement of the fill behind the wall, and so these might not represent equilibrium conditions for the wall heights shown. A reduction in the interface friction angle to around 26° does not seem to have much effect on the stability of Wall A, but at about this value the mode of failure changes from overturning to sliding. Further reductions in the angle are accompanied by a reduction in the safe height of the wall. Rather surprisingly, for Wall B a reduction in the interface angle from 45 to 44° affected its safe height: the reason for this sensitivity is not known. As the angle is reduced further to about 25°, displacement of the wall increases but overturning remains the mode of failure. Below this value, failure occurs by sliding and, as with Wall A, the safe height of the wall then reduces in line with the interface angle.

Sliding movements at the base can be readily identified from the plot of displacement over the height of a wall: for example as shown in Figure 10. The height-dependent component of displacement might be due to overturning: sliding can also occur between the blocks of a wall, but sliding movements in the walls at Kingstown would have been small.

For the circumstances at Kingstown, a value of about 25° is implausibly low for the interface block friction and also for the interface between the base of the wall and the foundation. Such a low value might be applicable for walls built of mudstone blocks prone to weathering, and ones built on particularly poor ground. In practice, the interface friction angle between the blocks might not be the same as that for the foundation.

#### *8.1.2 Corner radius*

To avoid numerical problems a nominal corner radius had to be assigned to the blocks. In most of the analyses a radius of 10 mm was adopted, but some analyses were undertaken with radii of 25 and 35 mm. This increase in radii might be taken to model the effects of weathering, and might better represent the blocks in many existing walls.

The results of some analyses are provided in Table 3. As can be seen, there was little effect of the radius on the performance of Wall A, but the results of Wall B suggest that the degree of roundness might be important for a wall close to collapse. This sensitivity is probably due to the increased susceptibility of the blocks to overturning.

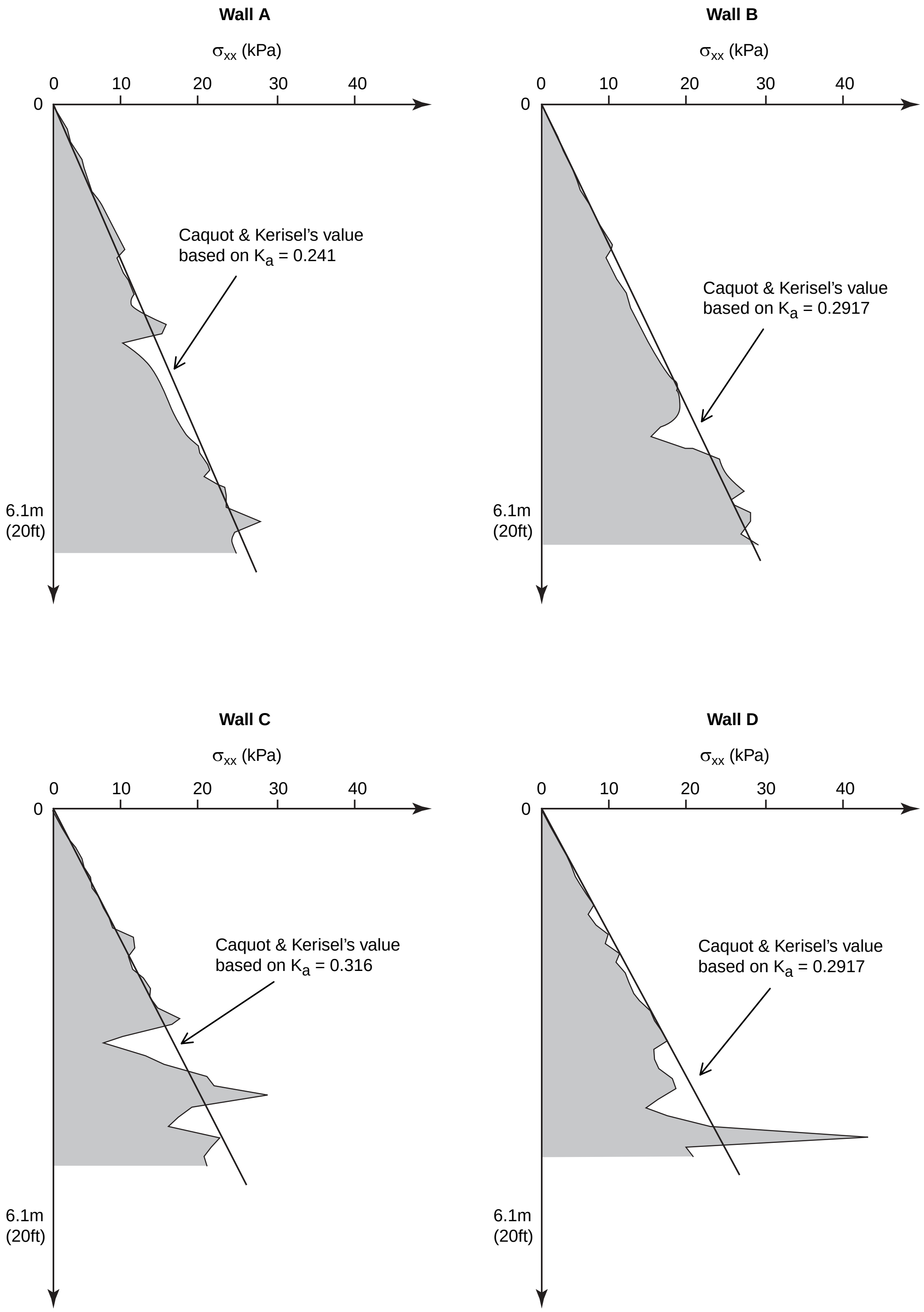



**Figure 7** Comparison of calculated $\sigma_{xx}$ distributions with those from Caquot and Kerisel's (1948) earth pressure coefficients, after Harkness *et al.* (2000)

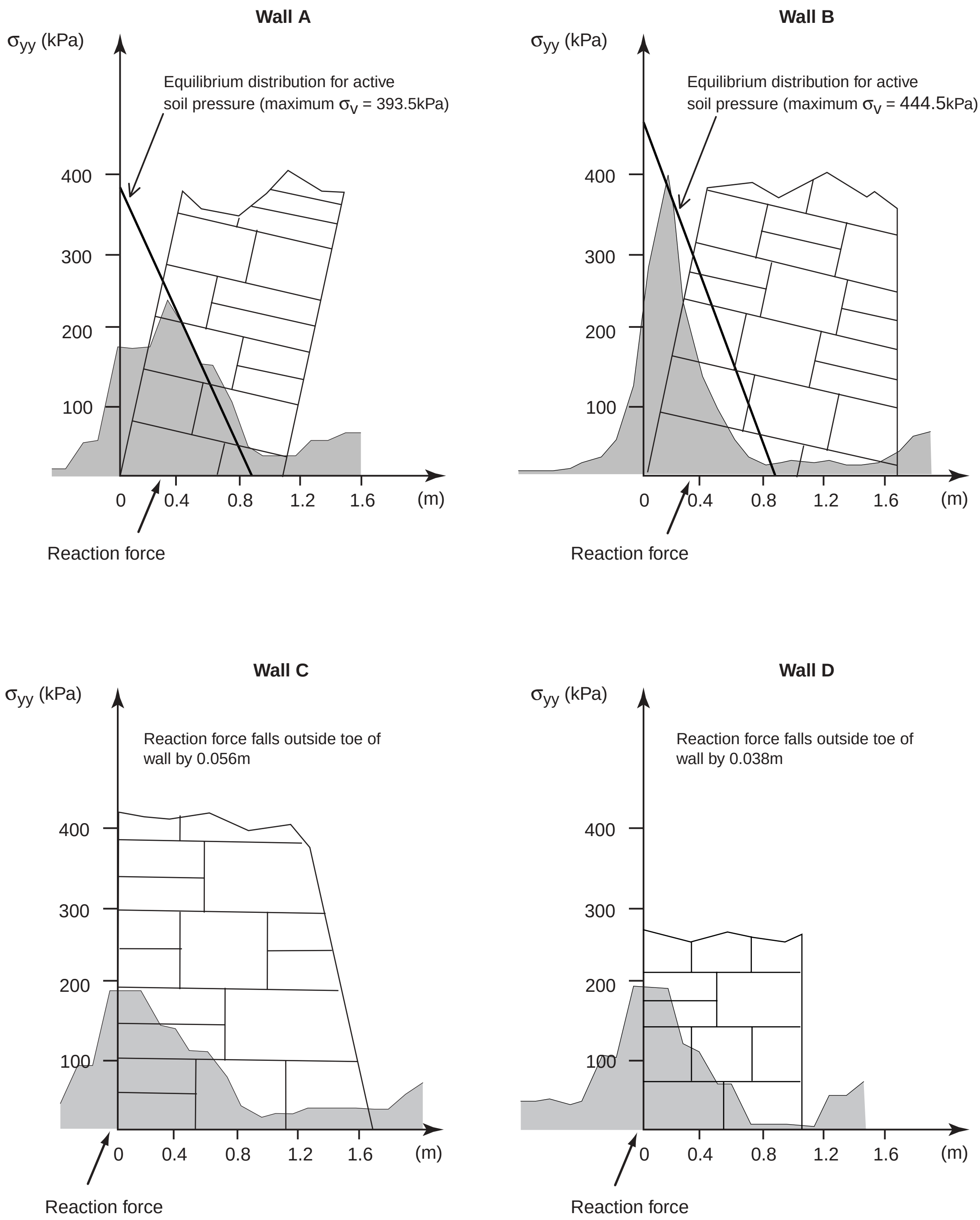



**Figure 8** Comparison of vertical stress distributions at the base of each wall calculated using numerical and limit equilibrium methods, after Harkness *et al.* (2000)

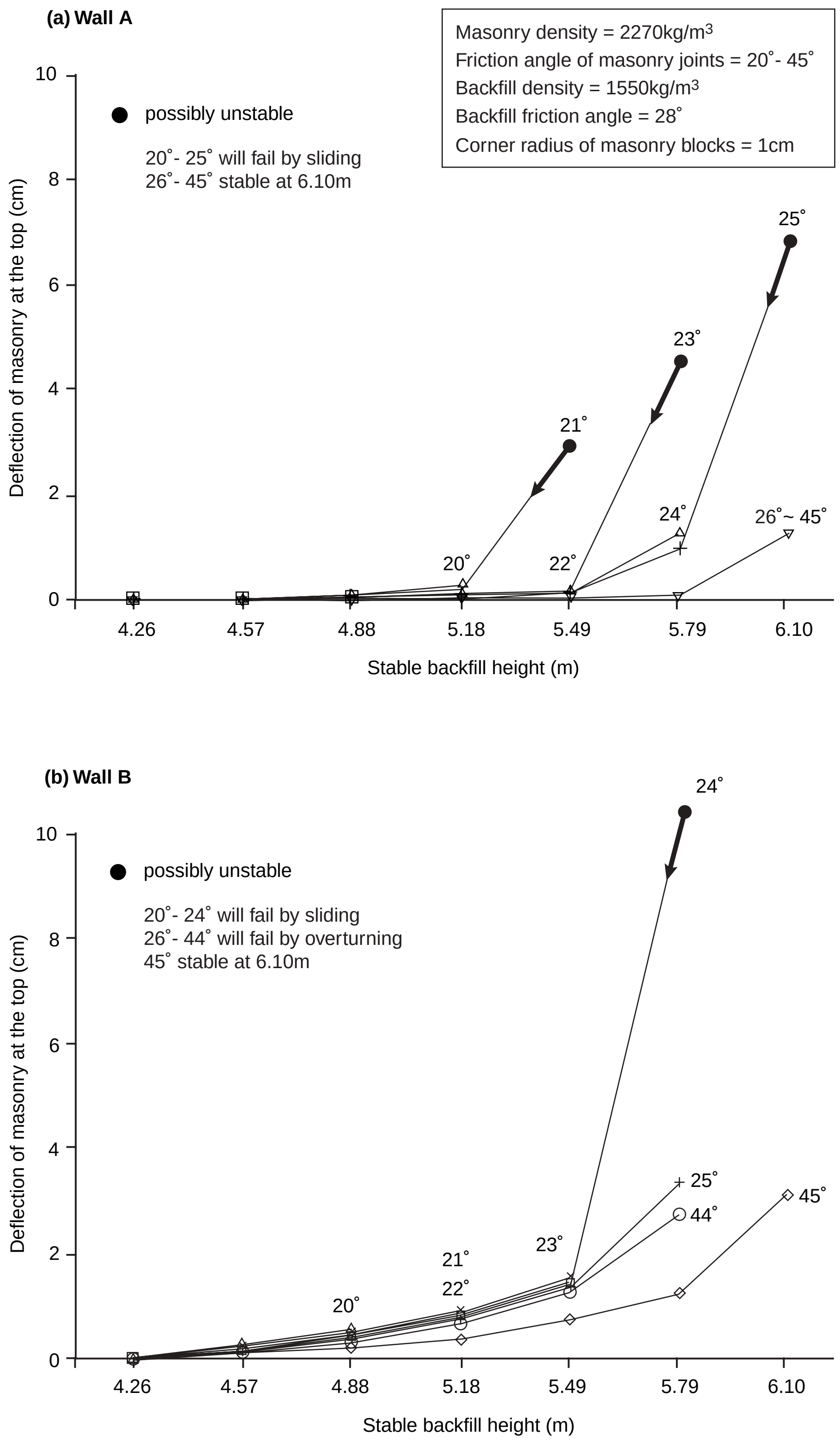



**Figure 9** Effects of friction angle of masonry joints on stability, from Harkness *et al.* (1998)

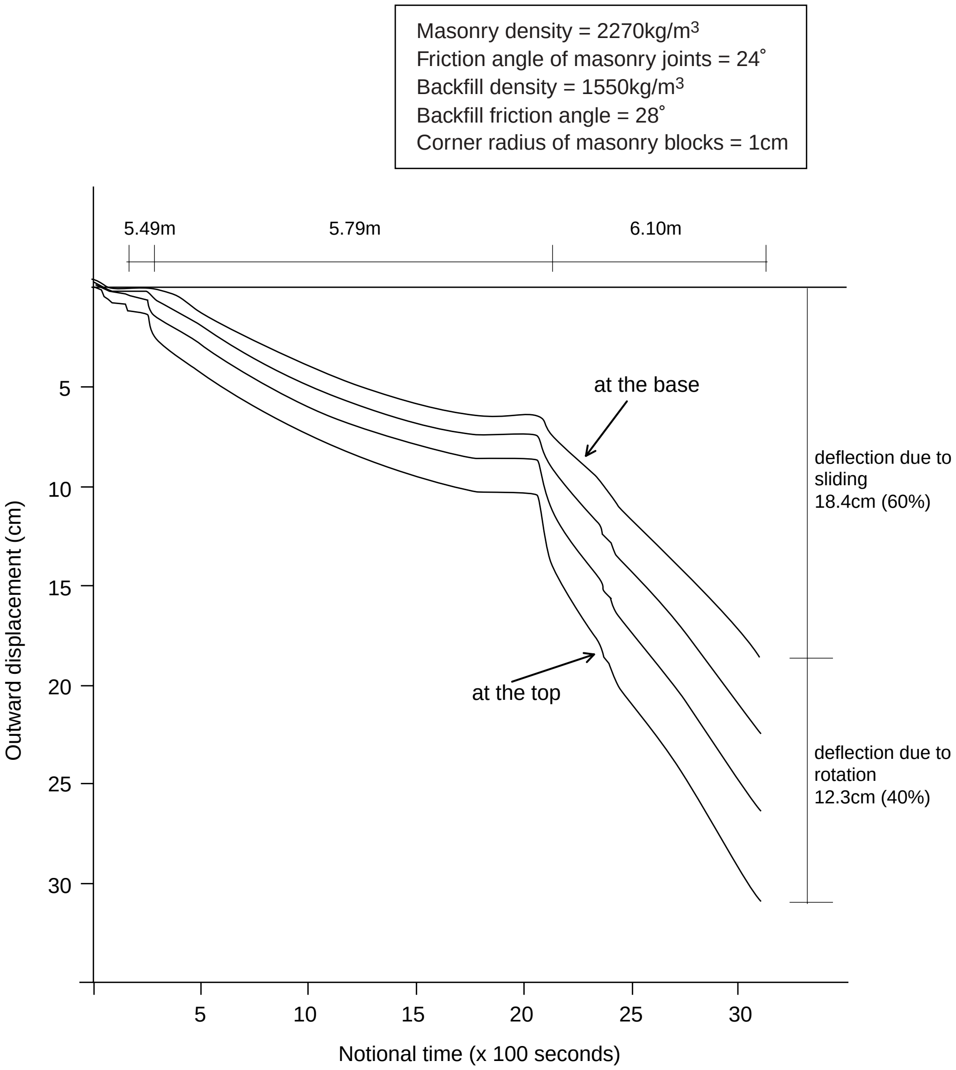
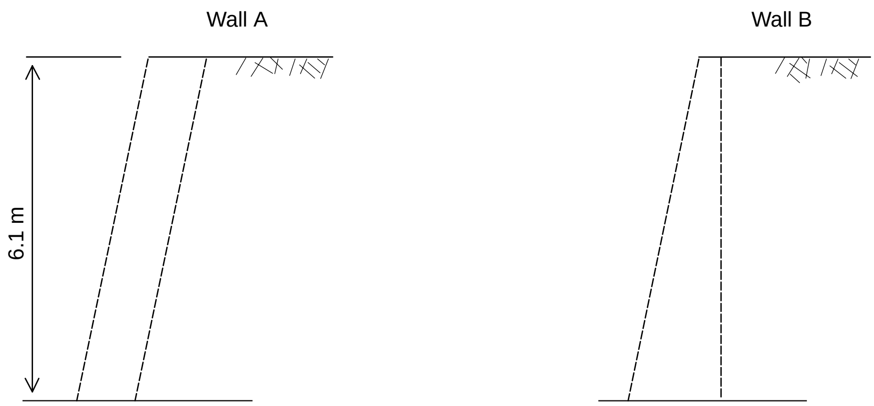



**Figure 10** Wall B: deformation of wall face, after Harkness *et al*. (1998)

**Table 3 Effect of corner radius on performance of Walls A and B**




*Variation of corner radius: δ (mm) = deflection of top of wall at last stable backfill height*

| | | Corner radius (mm) | | | Corner radius (mm) | | |
|---|---|---|---|---|---|---|---|
| *Backfill height (m)* | | *10* | *25* | *35* | *10* | *25* | *35* |
| 5.49 (18ft) | δ | 15 | 15 | 18 | 60 | 101 | collapsed |
| 5.18 (17ft) | δ | 5 | 5 | 6 | 16 | 17 | 19 |
| 4.88 (16ft) | δ | 3 | 3 | 3 | 9 | 9 | 10 |

*Properties common to these analyses:*

Masonry walls: Density = 2270 kg/m$^3$ Joint Friction = 45°

Backfill: Density = 1550 kg/m$^3$ Friction angle = 20°

***8.1.3 Joint pattern***

The results of all the analyses showed that the earth pressure acting on the back of all the four walls was close to active conditions; that is, close to the minimum attainable value. The overturning moments and shear forces acting on the walls would, therefore, be unaffected by the joint pattern. However the weight of the wall would be slightly affected by the number of joints.

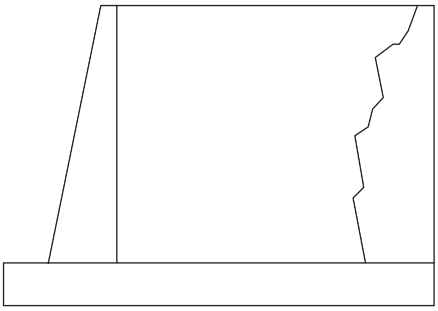
The various jointing patterns investigated are shown in Figures 11 and 12 for walls B and C respectively. Velocity vectors for some of the analyses undertaken on Walls B and C are shown in Figures 13 and 14 respectively: note that, because of the wide variation in movement, the vector scales on these figures are not the same. The movements of the wall face for these analyses are shown in Figures 15 and 16 respectively. A summary of the data at the end of the analyses is given in Figures 17 (Wall B) and 18 (Wall C) and Table 4. (Figures 11 to 16 are derived from Harkness *et al.*, 1998, or, in the main, from Brady and Kavanagh, 2001.)

The data show that the successive introduction of horizontal or sub-horizontal joints led to increasing movements of the wall face. However, for the range of input data used, sliding between the facing blocks did not significant affect the stability of Walls B and C. The magnitude and pattern of the movements would be a function of the shear stiffness of the interface between the blocks and the location and orientation of the joints. For a wall with a constant cross-section such movements are likely to manifest themselves as a uniform outward translation. On the other hand, for a wall with a tapering cross-section these movements might generate a convex shape to the face: as can be discerned from the data given in Table 4 for example.

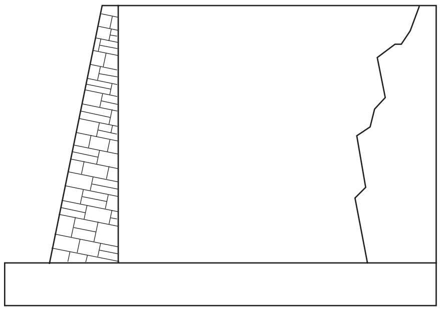
For the stable arrangements of Wall B, the results of the analyses show that the introduction of a sub-horizontal joint increases the outward movement at the top of the wall by only about 1 mm. The calculated outward movements for the various arrangements of Wall C are relatively high with the movement at the top of the wall for the monolith (C1) being about 44 mm: confirming that the wall was close to failure. The introduction of seven horizontal joints (C1 to C5) increased the outward movement at the top of Wall C to about 61 mm; that is, about 2.5 mm per joint. From comparison with Wall B, and as might be expected, the increase in outward deflection per joint decreased with increased inclination of the joint. Arrangement C2, with 18 horizontal joints (and, more importantly, some vertical joints) failed at a height of 4.9 m.

The introduction of two sub-vertical joints into wall B4 (that is to form B6) had little effect on the stability of the wall and, as shown in Table 4(a), the outward movement at the top of the wall increased marginally from 5.6 to 5.9 mm. But the wall became unstable with the introduction of additional sub-vertical joints: the difference between the results obtained for joint patterns B6 and B7 is quite marked. Similarly, the introduction of two vertical joints to joint pattern C4, to form C6, led to instability.

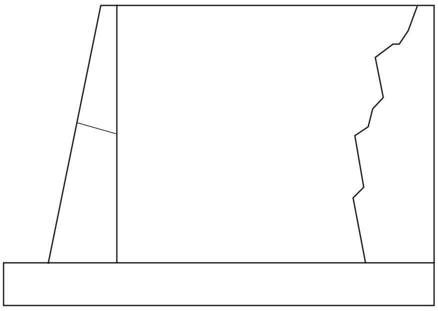
For a given set of disturbing forces, the effect of a horizontal, or nearly so, joint increases the outward deflection of the wall face but, unless this leads to geometric instability, it does not affect the stability of the wall. On the other hand, a vertical joint running parallel to the face can substantially affect stability: it introduces the



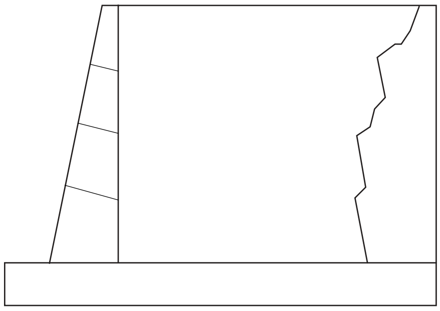
Joint pattern B1
0 joints



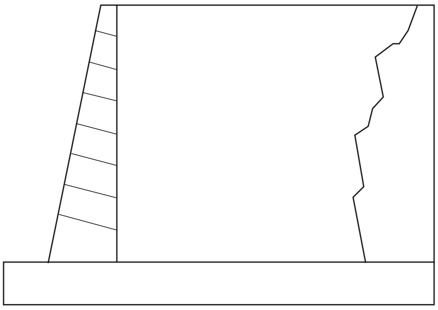
Joint pattern B2
as used to model wall at Kingstown



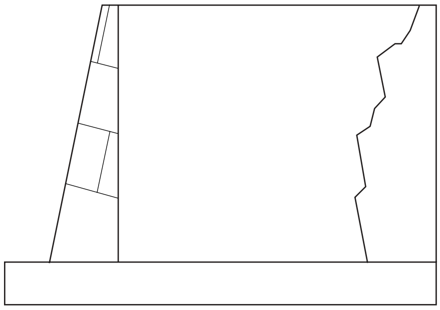
Joint pattern B3
1 sub-horizontal joint



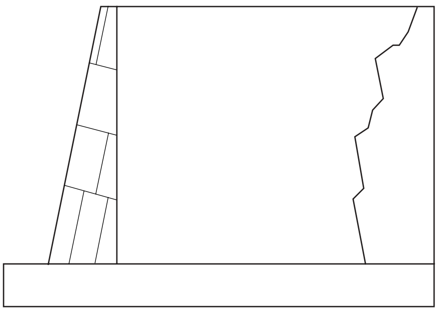
Joint pattern B4
3 sub-horizontal joint



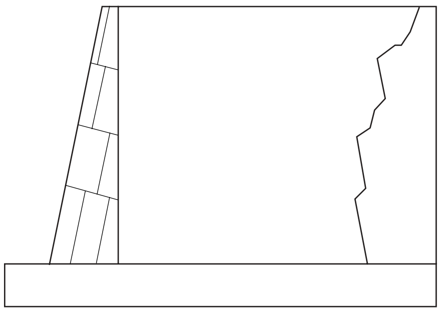
Joint pattern B5
7 sub-horizontal joints



Joint pattern B7
3 sub-horizontal joints
2 sub-vertical joints

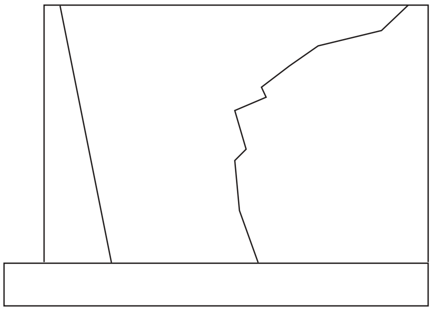


Joint pattern B7
3 sub-horizontal joints
4 sub-vertical joints

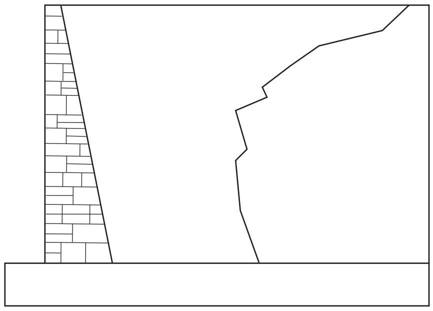


Joint pattern B7
3 sub-horizontal joints
5 sub-vertical joints

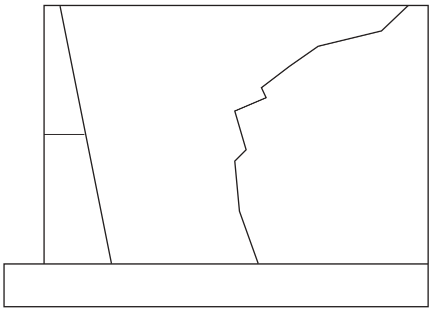
**Figure 11** Jointing patterns used in UDEC analyses for Wall B



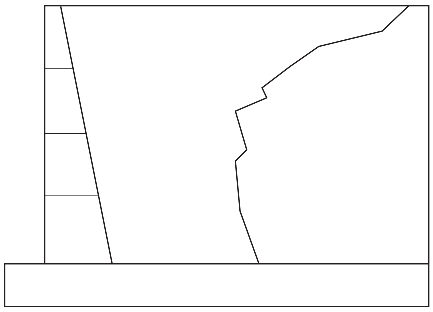
Joint pattern C1
0 joints



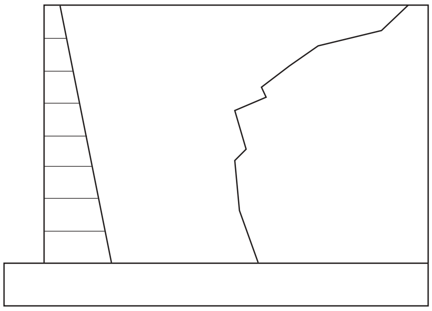
Joint pattern C2
as used to model wall at Kingstown



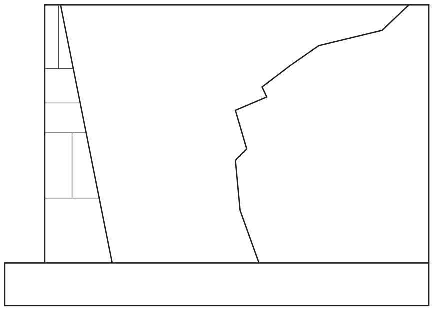
Joint pattern C3
1 horizontal joint



Joint pattern C4
3 horizontal joints

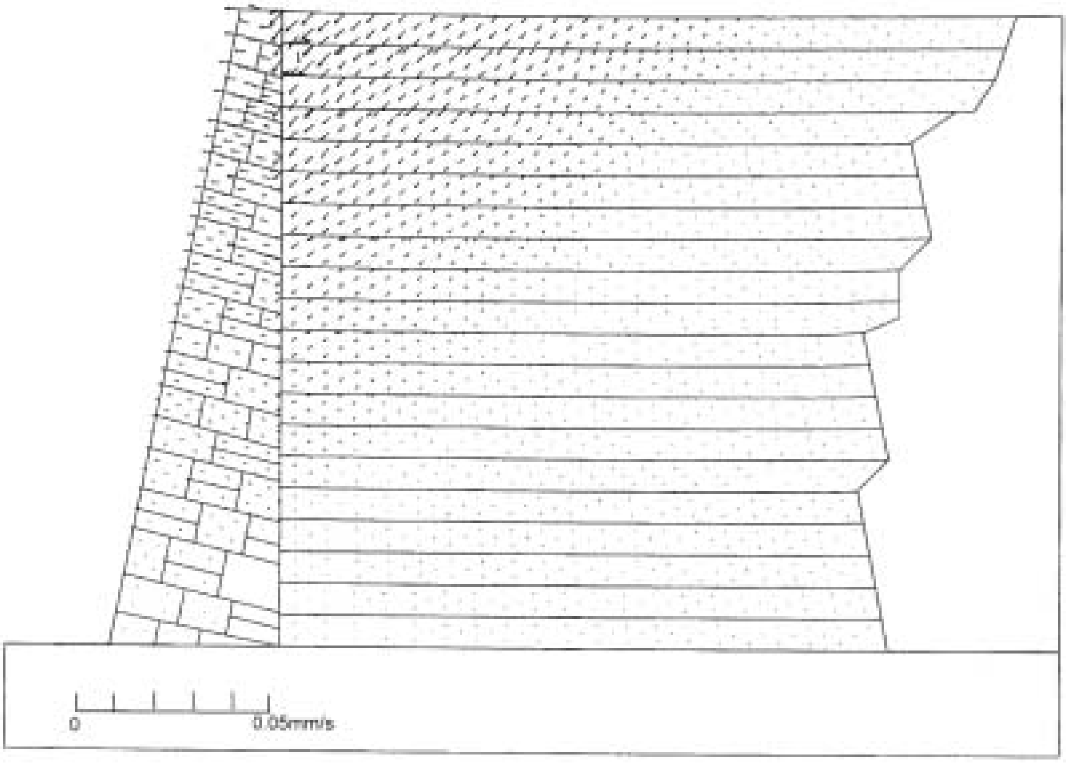


Joint pattern C5
7 horizontal joints

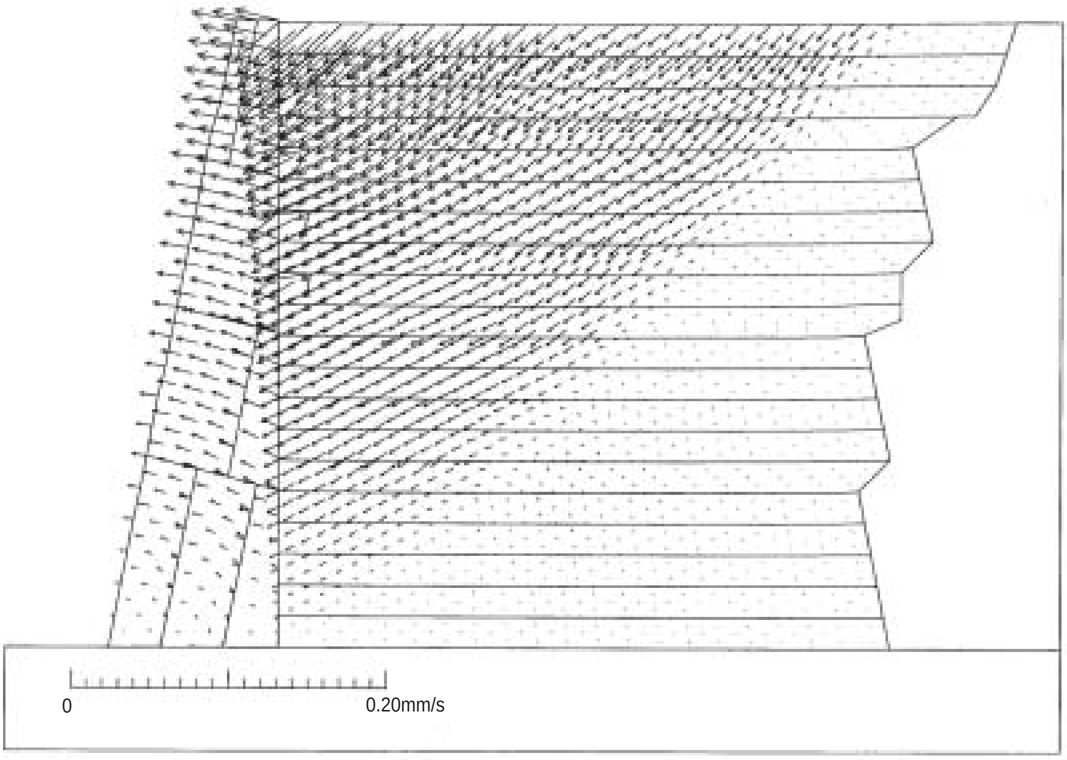


Joint pattern C6
3 horizontal joints
2 vertical joints

**Figure 12** Jointing patterns used in UDEC analyses for Wall C

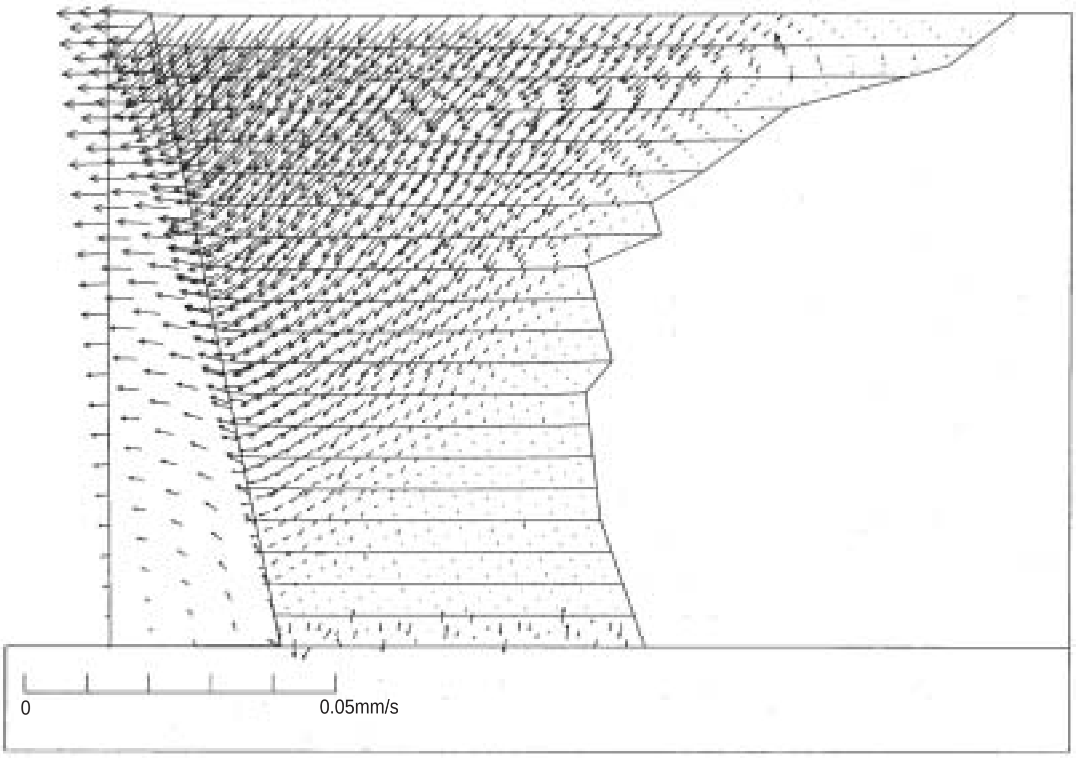



(a) Joint pattern B2: wall in equilibrium at end of construction

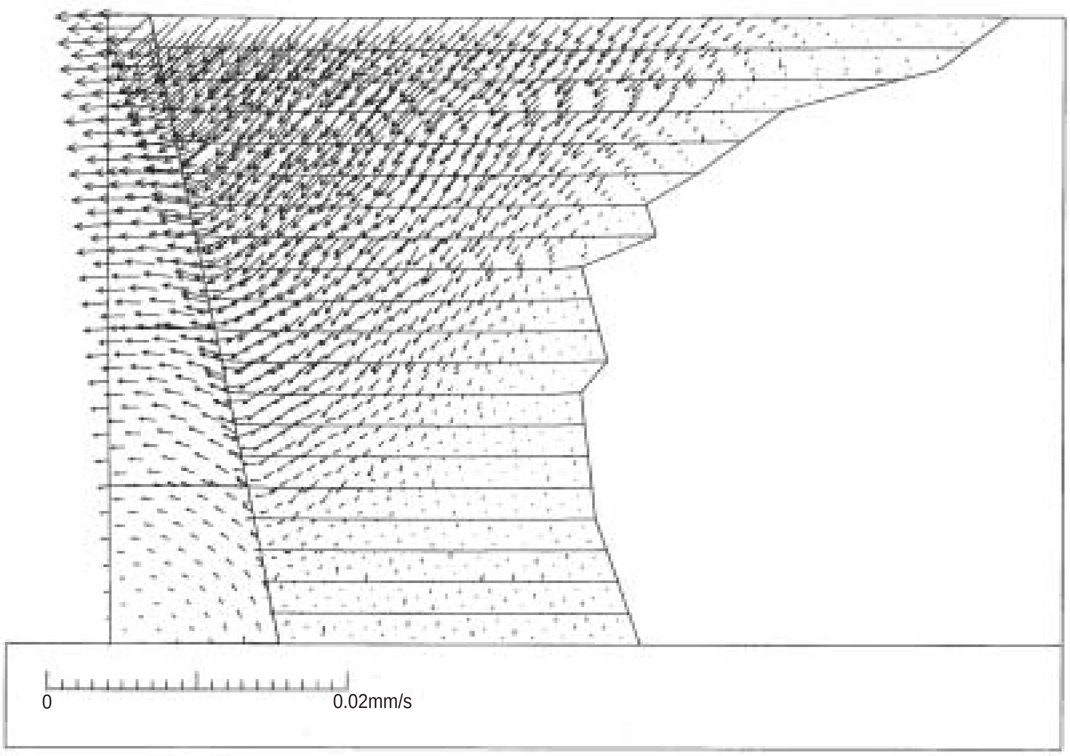



(b) Joint pattern B7: wall assumed to fail at stage 11

**Figure 13** Velocity vectors for Wall B

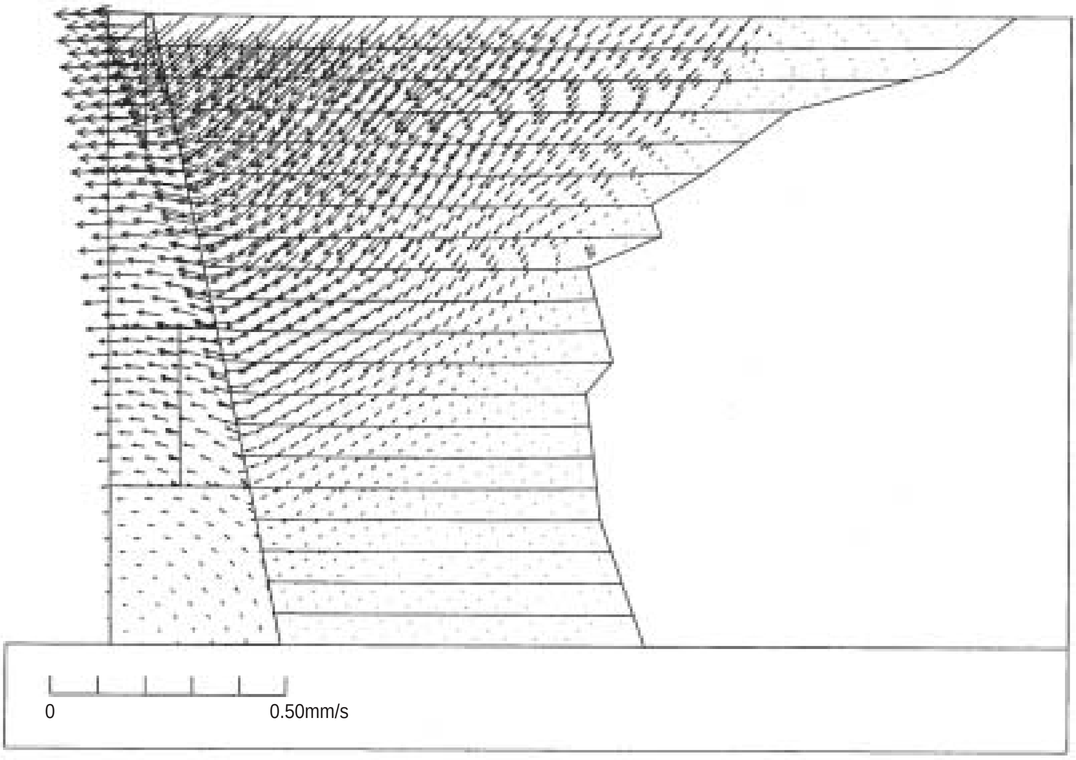



(a) Joint pattern C1: wall in equilibrium at end of construction



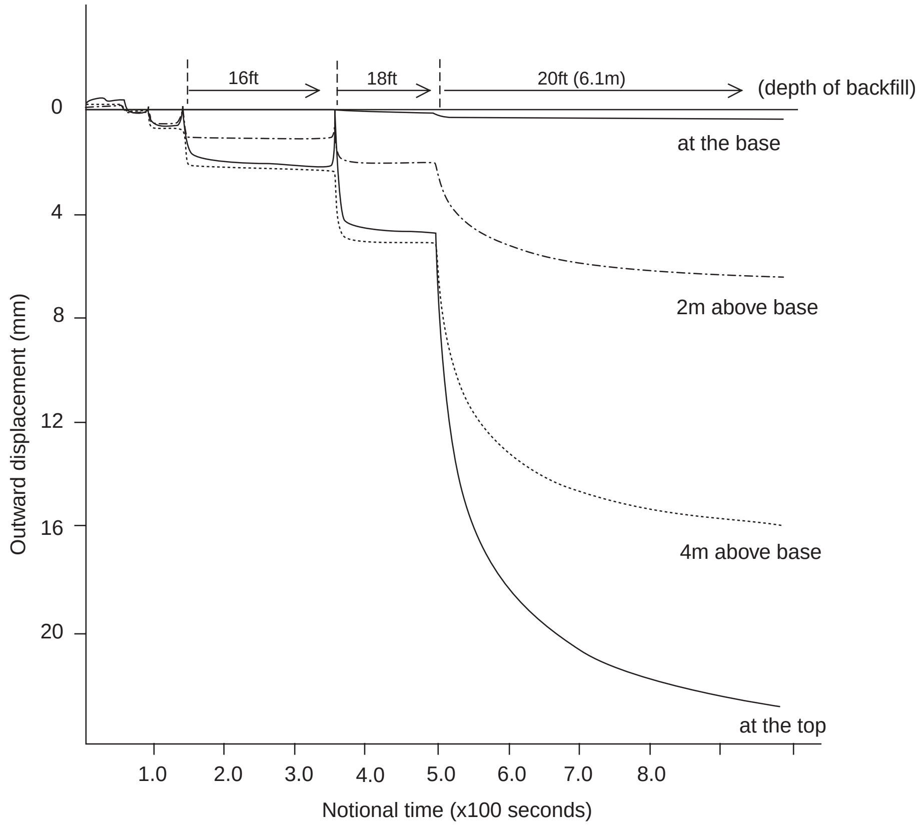

(b) Joint pattern C4: wall in equilibrium at end of construction

**Figure 14** Velocity vectors for Wall C

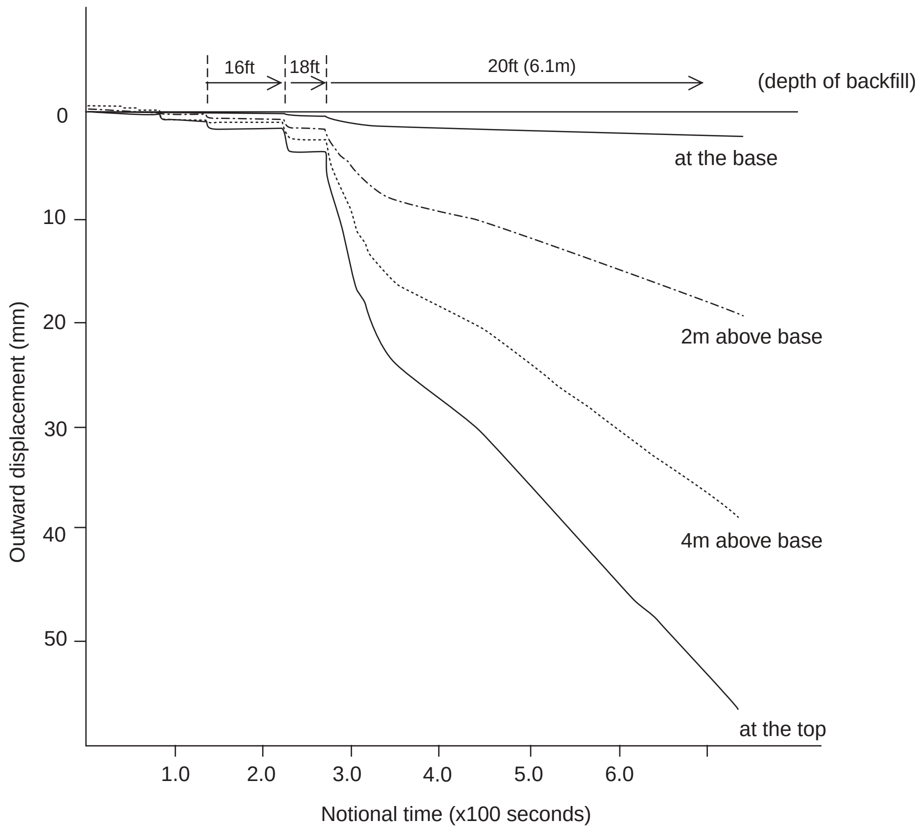



(c) Joint pattern C6: wall assumed to fail at stage 11

**Figure 14 (Continued)** Velocity vectors for Wall C

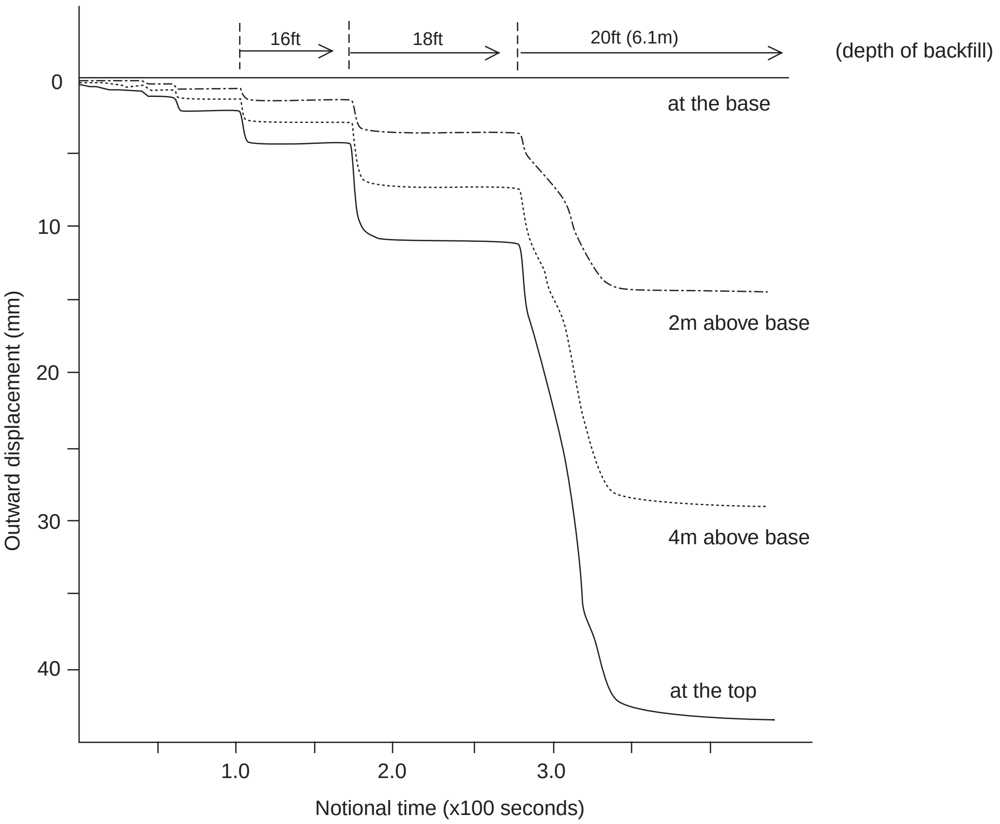



(a) Joint pattern B2

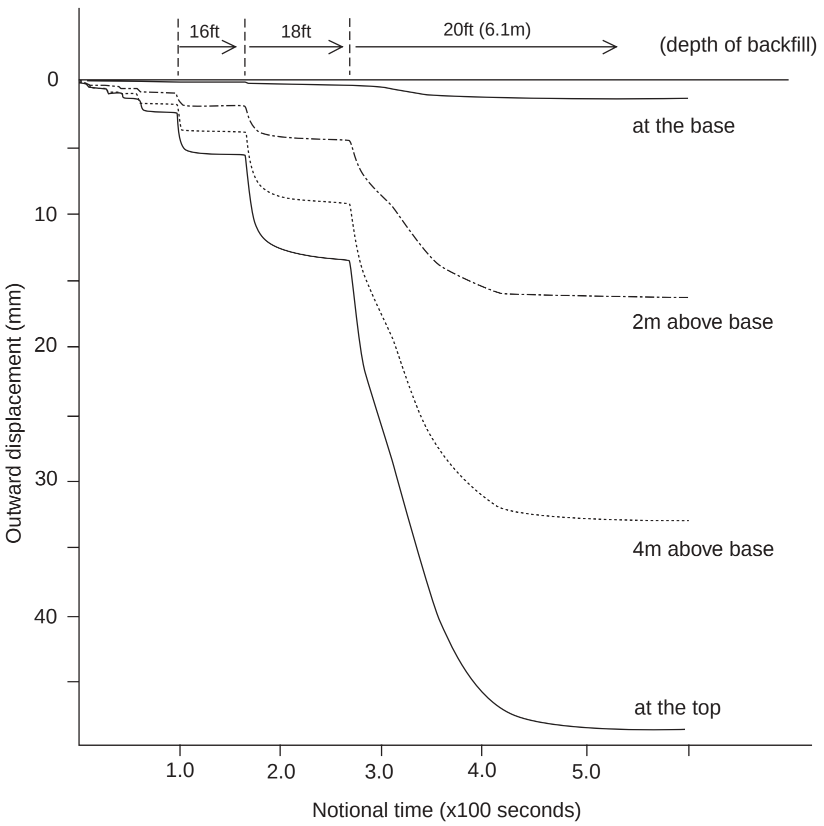



(b) Joint pattern B7

**Figure 15** Displacement of Wall B

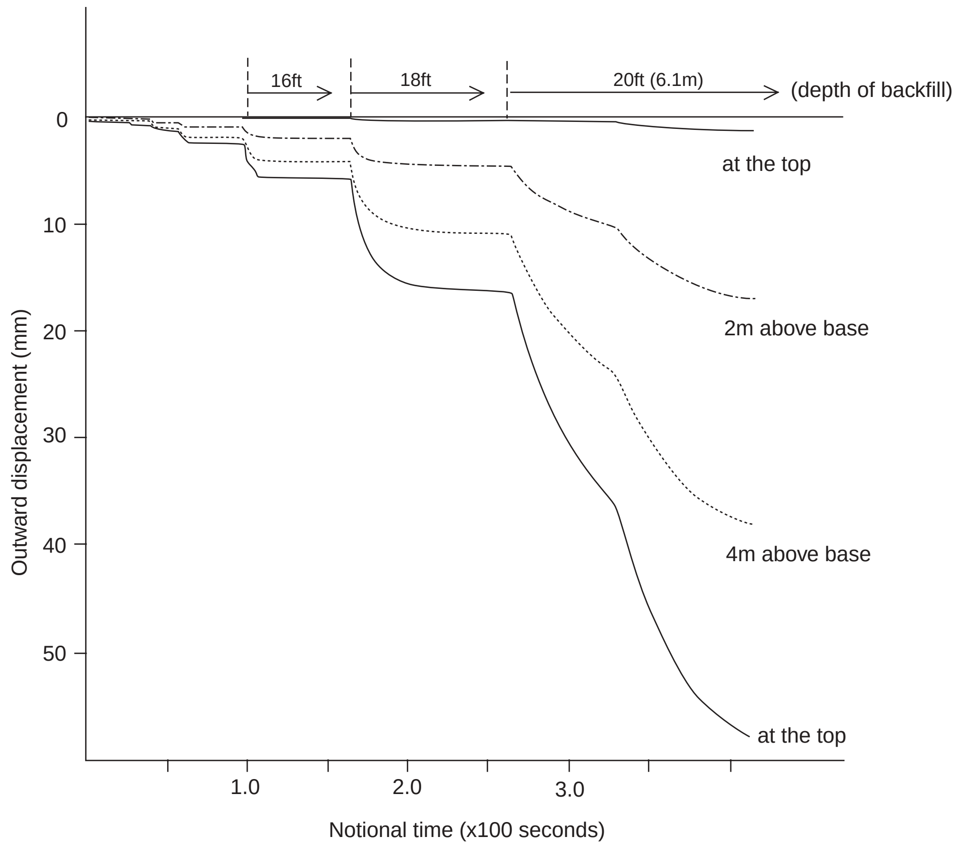



(a) Joint pattern C1




(b) Joint pattern C4

**Figure 16** Displacement of Wall C




(c) Joint pattern C6

**Figure 16 (Continued)** Displacement of Wall C

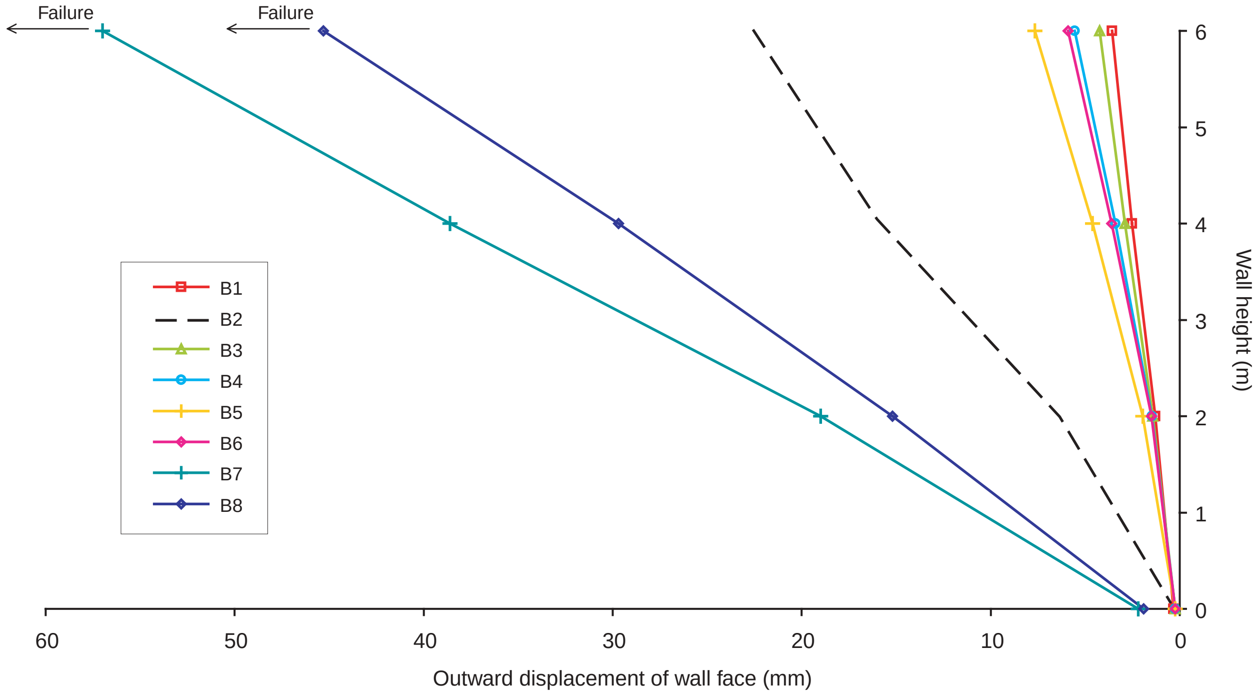



**Figure 17** Displacement of Wall B at end of analysis

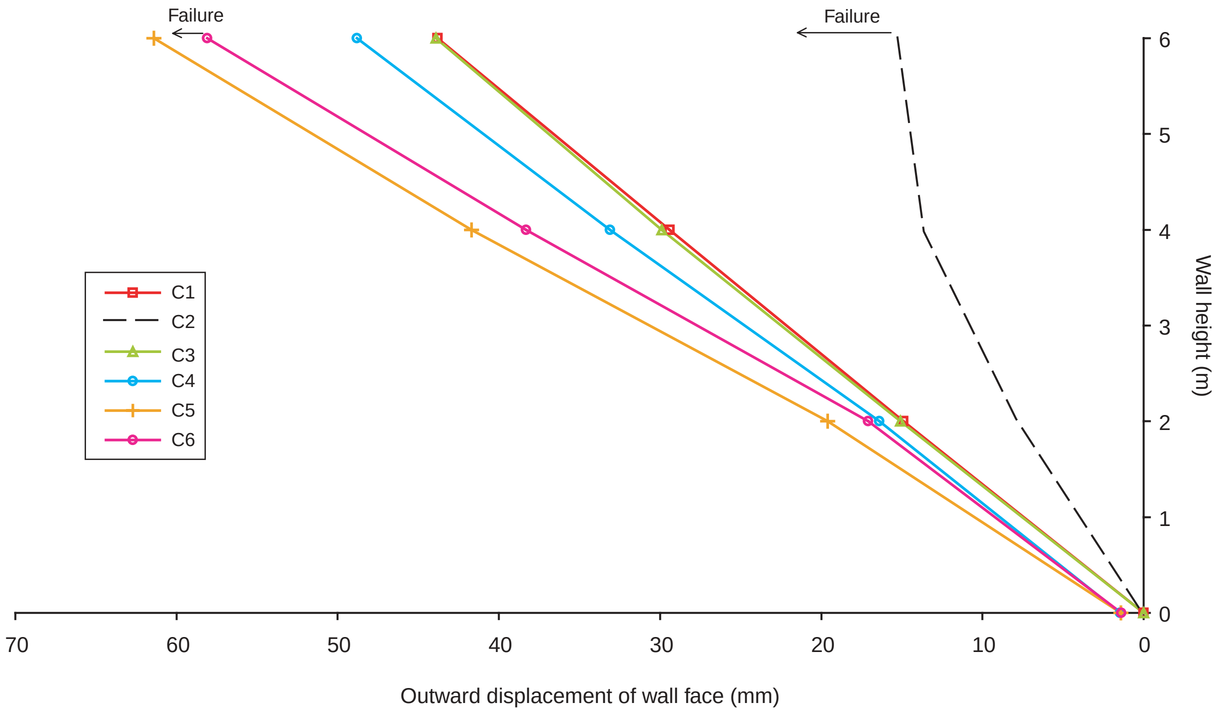



**Figure 18** Displacement of Wall C at end of analysis

**Table 4 Horizontal displacement at end of analyses**

| | Horizontal displacement (mm) | | | |
|---|---|---|---|---|
| | 1 (at the base) | 2 (2m above base) | 3 (4m above base) | 4 (at the top) |
| ***Joint pattern reference see Figure 9*** | | | | |
| B1 | 0.3 | 1.3 | 2.5 | 3.6 |
| B2 | 0.3 | 6.4 | 15.8 | 22.7 |
| B3 | 0.3 | 1.4 | 2.9 | 4.2 |
| B4 | 0.2 | 1.5 | 3.4 | 5.6 |
| B5 | 0.2 | 2.0 | 4.6 | 7.7 |
| B6 | 0.2 | 1.5 | 3.6 | 5.9 |
| B7 | Failure at stage 11 (20ft backfill) | | | |
| B8 | | | | |

(a) Wall B

| | | | | |
|---|---|---|---|---|
| ***Joint pattern reference see Figure 10*** | | | | |
| C1 | 0.0 | 14.9 | 29.4 | 43.8 |
| C2 | Failure at stage 9 (16ft backfill) | | | |
| C3 | 0.0 | 15.1 | 29.9 | 43.9 |
| C4 | 1.5 | 16.4 | 33.1 | 48.8 |
| C5 | 1.4 | 19.6 | 41.7 | 61.4 |
| C6 | Failure at stage 11 (20ft backfill) | | | |

(b) Wall C

possibility of local overturning of the blocks within the wall - the outward rotation of a block might lead to a cascade of other blocks local to it, and eventual collapse. This seems intuitively correct: an extreme analogy is the relative stability of a deck of cards placed face down, and on edge.

# 9 Discussion

### 9.1 Defining overturning safety

As shown in Section 5.4.1, the factor of safety against overturning depends on the method of calculation. For example, from Figure 2, the factor of safety calculated using $P_a$ for a backfill height of 5 m for Wall C is about 2, but for $P_a$ split into components $P_n$ and $P_p$ it is only about 1.45: a value which some might find unacceptably low. Yet the physical situation has not changed - the difference is merely a result of how the factor is calculated. Note that the relations between the factors of safety vary according to circumstance - for example, with the geometry of the wall. Apart from being a possible source of confusion, it leaves the question of just what minimum factor of safety is required for either method of calculation.

An approach based on the eccentricity of the resultant force acting on the base does not suffer this problem. The demand for the resultant to fall within the middle third of the base, so that tension is not generated along the base, is equivalent to a value of 3 for (b/2e). The data given in Figure 2 show that the adoption of this rule would limit the height of backfill to about 2.9 m, which is just over half the height at collapse. On this basis, it seems that the rule is overly conservative. Furthermore, as demonstrated below, the fact that a tension crack is not developed on the base of a block wall does not mean that one is not formed elsewhere. Where the resultant acted within the mid two-thirds of the base, the maximum eccentricity would be (b/3), and the minimum ratio of (b/2e) would be 1.5: at this point, the tension crack would be developed over half the width of the base of a monolith.

An approach based on the margin of safety does not vary according to the calculation method and can be used as part of a probability or reliability-based method of assessment. However it would be particularly difficult (if at all practical) to define the likely range of values for some of the input variables that govern the behaviour of a masonry-faced earth retaining wall. In addition, the margin must be compared with either the disturbing or resisting force in some way to define the probability of failure.

Note that the results of a numerical analysis have to be interpreted in some way to derive a measure of safety.

It is the case that all the methods used to assess stability against overturning do not have unrestricted applicability and are somewhat arbitrary - and so, therefore, are the minimum safety factors values prescribed in a design or assessment code. Thus the blind adoption or prescription of any particular method should be avoided.

### 9.2 Behaviour of block walls and monoliths

A Coulomb-type approach to the design or assessment of a block wall might overestimate the safety against overturning, no matter how safety is defined. The results of the UDEC analyses show that the presence of joints can have a significant effect on the stability of a wall. For example, according to Table 4, arrangement C2 failed at a height of about 4.9 m (16 feet) whilst the monolithic wall was stable at 6.1 m (20 feet). On this example alone, the assumption that a wall acts as a monolith would overestimate stability by a factor of around 1.25. The reason for the discrepancy is that overturning only occurs along the base of a monolith, but in a jointed wall it can occur on a plane inclined to the base. Because the wall has no tensile strength, the weight of the section of wall lying beneath the plane and towards the heel of a wall does not contribute to the overturning resistance of the wall. This accounts for the pattern of failure shown by the walls at Kingstown (various joint patterns), the unmortared brick walls built by Hope at Chatham (Anon, 1845), and it fits the findings of the model tests undertaken by Casimir Constable (1875). A limit equilibrium analysis of Wall C taking account of the inclination of the observed tension crack, or tearing plane, provides a safety factor of about unity. Thus in undertaking such an analysis it is necessary to take account of the inclination of this plane. The inclination is a function of the height and geometry of the wall, the shape of the blocks, the properties of the blocks, and the stability of the wall. For a wall close to collapse, a search for the critical plane might be best concentrated at an angle of 45° to the base. Because the disturbing forces acting to overturn a wall are also a function of the location of the plane, the plane might not pass through the toe of a wall and in complex conditions it might be necessary to identify its location through a numerical analysis. It is pertinent to note that Wong and Ho (1997) reported that the actual failure mechanism of the wall at Kwun Lung

Lau was complex and '*different to those assumed in the conventional slope and retaining wall analyses*': the actual plane seemed to exit the wall at about its mid-height.

The introduction of vertical joints running parallel to the face of a wall thus reduces the overturning resistance of the wall. And, as could be expected, the influence on performance of vertical joints decreases with the height above the base. Furthermore, and as shown by the later UDEC analyses (for the range of input variables used), the introduction of a vertical joint might not generate any substantial increase in outward deflection. At the risk of over-simplification, the introduction of a vertical joint either has no effect on stability or it precipitates collapse. In support of earlier comments, failure is, therefore, sudden and of a brittle type. This should not be surprising for an assembly of unbonded blocks: it has important ramifications for the inspection and assessment of in-service dry-stone walls, and for the use of materials and techniques for repairing and strengthening such walls.

As the overturning moment increases, the normal and shear stresses on the tearing plane become increasingly concentrated towards the point of overturning and the proportion of the plane over which no stress acts increases. In other words the length of the crack increases from the back to the wall towards the fulcrum. At the point of failure the stresses are concentrated over an infinitesimally small length. The appearance of such a crack might suggest itself as a plane of sliding, but it is a plane of tearing. In a plane strain analysis a dry stone wall has no coherence or resistance to tearing, but in practice some resistance might be generated along the line of a wall: thus, as mentioned previously, the bulging phenomena noted on many in-service walls requires a three-dimensional analysis. Although the length over which non-zero stresses are generated reduces with increasing overturning, the ratio of the normal and shear stress does not change and so the degree of overturning cannot be assessed by the ratio of these stresses (that is by the mobilised angle of friction). In practice, however, at some point the strength of the blocks will be exceeded and fracturing of the blocks will trigger a collapse: this might confuse a post-mortem of a collapse. It will be appreciated that the strength of a stone block is a function of its dimensions and shape - including the roundness of the corners.

### 9.3 Settlement

Overturning will only occur on a 'rigid' foundation. Of more general concern is the magnitude and direction of the resultant force acting on the foundation. This soil-structure interaction problem might be quite complex, but reliable solutions can be derived from numerical analyses. It is surprising that, nowadays, only rarely do textbooks provide solutions derived from numerical analyses alongside those obtained from other methods.

Clearly, the magnitude and distribution of the foundation pressures affect the ensuing settlement. Whilst not relevant to the walls at Kingstown, such pressures are likely to be of greater importance in the design of many walls than the factor of safety against overturning. The distribution of the vertical and horizontal stresses acting on the foundation cannot be defined through a limit equilibrium analysis, and so a pattern of distribution has to be assumed. The distribution could be based on that proposed by Meyerhof (1953), or a trapezoidal (where $b/2e > 3$) or triangular pattern (where $b/2e < 3$) could be assumed. It will be appreciated that the bearing capacity of a foundation is a function of both the vertical and horizontal stresses acting on it.

### 9.4 Assessment

A measure of the stability of an in-service structure could be based on the maximum deformation of the wall face. The results of the UDEC analyses suggest that the top of the walls at Kingstown could deflect about 100 mm, or about $1/60^{th}$ of their height (about 1° or so) without collapsing. However the maximum allowable deflection is likely to be a function, amongst other factors, of the thickness/height ratio of a wall and its geometry. Furthermore localised bulging rather than outward rotation is more likely to be seen on dry-stone retaining walls that have been in service for a hundred years or more. The presence of bulging is evidence that at some time or other the (local) factor of safety was unity. A number of mechanisms could be advanced for such distortions such as:

1 the local build-up of water pressure behind a wall where the drainage paths through it have become clogged with fines washed out of the backfill - the consequences of this might vary seasonally;

2 by analogy with masonry buildings, the 'loss of integrity of rubble cored wall' see Ashurst and Ashurst (1989);

3 seasonal movements within the backfill, similar to the ratcheting phenomena through cyclic temperature changes as described by England (1994);

4 fracturing of the blocks by, for example, frost action and impact forces;

5 the wedging apart of the blocks by the growth of tree roots;

6 foundation movements, which might be triggered by seasonal changes in weather.

Such mechanisms suggest that the threshold movement at the onset of a bulging failure is more likely to be of relatively constant magnitude rather than a proportion of, for example, wall height. Clearly, bulging cannot be predicted using an approach that treated the wall as a monolith, but it would also be difficult to model (predict, simulate or back-analyse) local bulging of an in-service wall using a numerical method of analysis. However it would seem useful to determine whether or not any rule-of-thumb can be derived from the results of a series of analyses. The emphasis here would be on establishing broad geometric rules rather than a detailed investigation of the effect of the various input data on performance.

### 9.5 Application of numerical methods to masonry-faced retaining walls

The results of the analyses reported by Harkness *et al.* (2000) were in good agreement with the performance of the walls built at Kingstown. Leaving aside the problem of modelling dilatant materials, there is every reason to suppose that a numerical approach could, with the correct input data, reproduce the behaviour of other walls. But the problem is to obtain the correct input data: this includes the properties of the backfill and foundation soil, the properties of the blocks making up the facing, the geometry of the wall, and the pattern of jointing in the wall.

The properties of the backfill might be obtained through a site investigation, but it will be difficult to obtain undisturbed samples of coarse-grained backfill - as commonly found behind dry-stone walls - and interpretation of site tests is problematic in such materials. Although the properties of the material making up the facing blocks can be estimated from site or laboratory tests, these do not have a dominant effect on the performance of a wall. Of more relevance is the interface angle of friction between the blocks, and this might be difficult to determine where the blocks are weathered and/or where fine particles have collected between them. It might also be difficult to determine the degree of roundness of the blocks within a wall. Even if the distribution were known, there remains a problem of assigning a value for a DEA analysis - a Monte-Carlo approach might be used to assign values to the blocks in a particular mesh. The geometry of a wall might be defined reasonably well using Ground Penetration Radar (GPR), see for example Kavanagh *et al.* (1999), but it might be difficult to determine the dividing line between the back of the wall and the coarse-grained backfill lying against the back of it. Finally, it would seem well nigh impossible to reproduce the actual jointing pattern in most in-service structures.

This rather daunting list of problems might lead to the conclusion that numerical methods are of little, if any, practical use for analysing the stability of block walls. However it should be appreciated that they are common to any method of analysis that seeks to determine the performance of a masonry-faced wall in close detail: the question is whether or not such detail is required. As discussed in Section 4, given an appropriate inspection regime there is no pressing need to quantify the stability of a structure that shows no sign of distress, and so it is likely that quantitative assessments would only be required for marginally stable structures. As mentioned earlier, although the input data can be tuned to reflect this condition, it does not validate these data or the prediction. Nonetheless, it does help to narrow down the range for some variables and this might help when considering the design of strengthening measures. In selecting the method of analysis, consideration will be given to the applicability, ease of use and cost of the methods available and also to the required accuracy of a solution. However, in retrospect, at present the applicability of a numerical method of analysis for assessing the stability of in-service dry stone walls would not seem to be justified for other than rare exceptions - such as the back-analysis of a failure as at Kwun Lung Lau. Such methods have more immediate appeal for other types of structure that have little resistance to tensile forces, such as masonry arches.

The limitations of numerical methods for assessment do not apply to anything like the same extent when it comes to the design of new structures. And there does seem a need for further analyses to better quantify the effect that some variables have on the performance of dry-stone walls and their derivatives. As mentioned earlier, there would be merit in determining the distribution of stresses acting on the base of walls founded on soils of varying stiffness (both short- and long-term values): current numerical methods are quite capable of producing reliable data of direct use to designers. Such sensitivity studies should help define the relative importance of the variables and in this way direct where efforts should be concentrated when gathering data for design. Although few dry-stone walls are built nowadays, the results of such studies would be of much wider interest.

The use of numerical methods will inevitably increase in line with the reduction in the cost of computing power. It can be expected that the use of some form of numerical modelling will be commonplace, if not the norm, as the turn-round for a numerical analysis becomes only marginally longer than required for completing a limit equilibrium analysis by hand. (This itself poses education and training problems for the industry, but such issues are outside the scope of this report.)

**9.6 Other issues**

The results of the UDEC analyses showed that the interface force acting on the back of the walls at Kingstown approached if not equalled that due to full soil friction. This stabilising force is commonly neglected in the design of new retaining structures - as is soil cohesion (and/or suction). Their omission from an analysis explains in large measure why many in-service masonry-faced earth retaining walls can be assessed as unstable when the reality is that they are performing satisfactorily. However the walls at Kingstown were constructed on a rigid foundation, and further consideration should be given to the effect of settlement of the wall on the development of the interface force.

Some of the findings of the analyses might only confirm, quantitatively, what masons knew by intuition or took to be common sense regarding the size and orientation of blocks in a wall. For example:

1 the largest blocks should be placed at the base with their shortest dimension in the vertical plane;

2 a backwards inclination of the joints between blocks, as shown in Figure 2 for Walls A and B at Kingstown, improves stability;

3 the insertion of through stones, see Jones (1990), improves stability by restricting the length of a vertical crack that can develop through a wall, and thereby limiting the mass of masonry that might otherwise not provide resistance to overturning;

4 stability can be improved by providing some interlocking of stone along a wall; that is, by providing a three-dimensional structure.

As a matter of course, maintenance and strengthening works should be properly targeted to the perceived problem at hand. One of a number of solutions might be used to increase stability against overturning; these include,

1 grouting up the joints in a dry-stone wall so that it would behave more or less as a monolith;

2 grouting up the retained soil to reduce lateral pressures,

3 installing bolts or anchors to tie the blocks of the wall together; these might extend into the backfill to form a nailed or anchored structure;

4 thickening the face of the wall, by spraying concrete or, better, by bonding new masonry facing units to the existing facing.

Such works might only need to be undertaken towards the base of a wall.

## 10 Conclusion

Surveys of 1474.5 km of the trunk road system in five agent authority areas in England and Wales have shown that there are 92.7 km of masonry-faced retaining walls on these roads with an estimated replacement cost of about £107 M. On average about £0.8 M is spent annually on the upkeep of these walls or just about 0.75 per cent of the cost of their replacement. The returns from a partial inventory indicate that there are some 9000 ± 2000 km of walls on the road system of Great Britain with a replacement cost of about £7.2 ± 1.6 billion. Of this, up to about 85 per cent is due to masonry-faced walls.

This stock is, by and large, performing satisfactorily although some structures are undergoing a process of gradual decline. At present, the stability of these walls is assessed qualitatively on the basis of their condition and of the significance of any defects in them: this approach is eminently sensible and cost effective. Current design codes preclude the inclusion of many of the optimistic but nevertheless real factors that explain the survival of the current stock of dry-stone walls and their derivatives. It is the case that a goodly proportion of the stock would not satisfy the requirements for stability as defined in current codes even though, in the main, their behaviour in service is perfectly adequate.

Given the low annual expenditure on maintenance and renewal, improvements in the current methods of assessment are unlikely to provide a dramatic reduction in expenditure. However, better methods should help direct resources to critical structures and prevent perfectly adequate structures from being strengthened or, in the extreme, replaced.

Although a limit equilibrium analysis can be used to calculate a factor of safety against overturning of a retaining wall, the value of the factor depends on its method of calculation. The methods used to assess overturning stability do not have unrestricted applicability and are somewhat arbitrary - and, therefore, so are the minimum values prescribed for them.

The assumption that a jointed wall acts as a monolith up to the point of collapse is non-conservative. The reason for this is that overturning can only occur over the base of a monolith, but in a jointed wall it can occur on a plane inclined to the base so that the full weight of the wall does not resist the overturning forces. This possibility must, therefore, be taken into account in a limit equilibrium analysis.

The essential behaviour of the four full-scale walls built by Burgoyne (1853) was reproduced in discrete element analyses undertaken using the program UDEC (Itasca, 1993). The results of the analysis confirmed, reasonably well, the values of the input properties assumed for the backfill and, to a lesser extent, those used for the blocks making up the wall. They also confirm that (a) full friction was developed between the back of the wall and the backfill and (b) active pressure conditions were developed in the backfill. Such optimistic assumptions might not be invoked in a limit equilibrium analysis. Analyses were undertaken to investigate the effect of varying some of the properties of the backfill and the blocks and the jointing pattern between the blocks. For the arrangements analysed (a) the introduction of a horizontal joint led to a small increase in outward deflection of the wall face - much as anticipated and (b) the introduction of a vertical joint either had very little effect on stability or it precipitated collapse - by reducing the overturning resistance. The brittleness of dry-stone walls has important implications for the inspection, assessment and maintenance of such structures.

At present, it is unlikely that the use of UDEC, or any other numerical method of analysis, would be justified for routine assessment purposes. However the use of numerical methods will increase with the inevitable increase in the availability of cheap computer power. Furthermore, modern analytical methods have an important role to play in developing and sharpening the criteria used in the assessment of the existing stock of structures. There is a need to develop assessment codes for existing structures to parallel the current codes that, with few exceptions, only address the design and construction of new works. It would seem entirely practical and economic to use numerical methods to define the interaction of the forces acting on the back of a retaining wall and on the foundation to the wall. The results from such analyses could be used to determine a factor of safety against overturning (if so required) as well as providing an estimate of the likely settlement and outward lean of the wall: not something that can necessarily be done with any other approach.

Given the estimated value of the stock of masonry-faced earth retaining walls in Great Britain it would not seem difficult to justify further work to improve methods of assessment and to provide a better understanding of the phenomenon of bulging. There seems to be no insurmountable practical or economic reason why the use of numerical methods for design and assessment should not, in the fullness of time, displace the use of other methods of analysis for the routine design and assessment of a range of geotechnical structures. The only question is how could this new era be promoted? That, in itself, requires further consideration.

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## Abstract

This report summarises the results of surveys of the stock of masonry-faced earth retaining walls along the highway network in the UK, providing estimates of their replacement value and annual maintenance expenditure. The current level of expenditure shows that the stock is performing well and that much of it has a considerable residual life. Despite this satisfactory position, many walls would not have an adequate factor of safety as required by current design codes. The report goes on to review the factors that affect the stability of such walls, and the methods used to characterize safety. It then describes and discusses the results of numerical analyses undertaken using UDEC, a discrete element program, on four full-scale dry-stone retaining walls built by Burgoyne at Kingstown, now Dun Laoghaire, in Ireland in 1834. The results of these analyses show that a conventional wedge-type analysis can overestimate the overturning resistance of a block wall when it is treated as a monolith. This is because vertical joints running parallel to the face of a wall allow tension cracks to develop within it so that not all the weight of the wall contributes to its overturning resistance. Analysis also shows that conventional measures of overturning stability do not have unrestricted applicability and are rather arbitrary and so, therefore, are the minimum values prescribed for them. Recommendations for further applications of numerical methods to aid designers and assessors of retaining walls are provided in the report.

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